J. Math. Kyoto Univ. (JMKYAZ)
36-4 (1996) 869-875

Relative entropy and mixing properties of interacting particle systems

By

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1. Introduction

Consider a Markov semigroup S_t on a compact metric space X that has the Feller property. If we start the process with some initial distribution μ, then it is not always true that for large t the distribution $\mu_t = \mu S_t$ is close to an invariant distribution for the process. While any limit point as $T \to \infty$ of the time average $\frac{1}{T}\int_0^t \mu_t dt$ is always an invariant measure, the same can not be claimed for limit points of μ_t itself. The simplest examples are provided by deterministic flows. However for any Markov chain on a finite state space, continuous time rules out periodic behavior and μS_t has a limit as $t \to \infty$ and this is always an invariant measure.

The natural question that arises is to determine if under some suitable conditions on the Markov semigroup S_t one can still claim that all possible limit points of μS_t are invariant measures. Such a result in conjunction with a uniqueness theorem for invariant measures will establish the convergence of μS_t to the unique invariant measure giving us a mixing result.

It has been conjectured that in the context of interacting particle systems the answer is in the affirmative under some very mild restrictions. Let $X = F^{\mathbf{Z}^d}$, where F is a finite set. The state η of the system is described by its values $\eta(x)$ for $x \in \mathbf{Z}^d$. The infinitesimal generator of the particle system is given by

$$\Omega f(\eta) = \sum_{T \subset \mathbf{Z}^d} \int_{F^T} c_T\ (d\xi, \eta)\ (f(\eta^\xi) - f(\eta)) \tag{1}$$

where the summation runs over all finite subsets of $\mathbf{Z}^d$. Here $c_T(\mathrm{d}\xi, \eta)$ describes the rates for Poisson events that change the current configuration η to a new configuration η^ξ that has been altered on the finite index set $T \subset \mathbf{Z}^d$ from η to ξ. A whole family of such Poisson events are taking place simultaneously and the infinitesimal generator reflects that. Of course a whole lot of these $c_T(\cdot,\cdot)$ may be 0. We say that a particle system has bounded flip rates if there is a bound on the sum of all the Poisson rates that could affect a loca-

Communicated by Prof. S. Watanabe, March 22, 1996

Supported by NSF grant 9503419 and ARO grant DAAL03-92-G-0317