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On ill-posedness and a Tikhonov regularization for a multidimensional inverse hyperbolic problem

By

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§1. Introduction

We consider an initial/boundary value problem for a hyperbolic equation:

$$(1.1)\qquad \begin{cases} u''(x,t)=\Delta u(x,t)+\lambda(t)f(x) & (x\in\Omega,\ 0<t<T) \\ u(x,0)=u'(x,0)=0 & (x\in\Omega) \\ u(x,t)=0 & (x\in\partial\Omega,\ 0<t<T). \end{cases}$$

Here $r\geq 2$ and $\Omega\subset\mathbf{R}^r$ is a bounded domain with smooth boundary $\partial\Omega$, $T>0$, and we set $u'(x,t)=\frac{\partial u}{\partial t}(x,t)$, $u''(x,t)=\frac{\partial^2 u}{\partial t^2}(x,t)$, and Δ is the Laplacian. Henceforth we always assume

$$(1.2)\qquad \lambda(0)\neq 0,\quad \lambda\in C^1[0,T].$$

Let $L^2(\Omega)$ be the space of all real-valued square integrable functions with the inner probuct $(\cdot,\cdot)_{L^2(\Omega)}$ and the norm $\|\cdot\|_{L^2(\Omega)}$. Let us denote the Sobolev space of order $s>0$ by $H^s(\Omega)$ (e. g. Lions and Magenes [13]). Under the assumption (1.2), for any $f\in L^2(\Omega)$, there exists a unique solution $u=u(f)$ to (1.1) such that

$$u=u(f)\in C^1([0,T];H_0^1(\Omega))\cap C^2([0,T];L^2(\Omega))$$

and

$$\frac{\partial u(f)}{\partial n}\in H^1(0,T;L^2(\partial\Omega))$$

(Lasiecka, Lions and Triggiani [10, Theorem 2.1] and the argument in §4 of Yamamoto[24]).

The term $\lambda(t)f(x)$ is considered an external force causing a vibration. We assume that λ is a known non-zero C^1-function and is independent of the space variable x, and $f\in L^2(\Omega)$ is unknown. We discuss

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