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On Kallianpur-Robbins law for fractional Brownian motion

Dedicated to Professor Hiroshi Kunita on the occasion of his 60th birthday

By

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1. Introduction

Let $\{B_t\}_{t\geq 0}$ be a d-dimensional Brownian motion and $V(x)$ be a summable function on $\mathbf{R}^d$ such that

$$\bar{V}:=\int_{\mathbf{R}^d} V(x)dx>0.$$

Then the random variables of the form $\int_0^t V(B_u)\,du$ are called the *occupation times* and the following theorem is well known as the Kallianpur-Robbins law.

Theorem A ([6]).

$$(d=1) \qquad \lim_{t\to\infty} P\left[\frac{1}{\bar{V}\sqrt{t}}\int_0^t V(B_u)du<x\right]=\sqrt{\frac{2}{\pi}}\int_0^x e^{-y^2/2}\,dy,\ x>0.$$

$$(d=2) \qquad \lim_{t\to\infty} P\left[\frac{2\pi}{\bar{V}\log t}\int_0^t V(B_u)\,du<x\right]=1-e^{-x},\ x>0.$$

This theorem was extended greatly by Darling-Kac ([4]) as follows and has been stimulated the interest of many authors for a long time. (See e.g. [7], [10]. See also H. Kesten [8].) Let $\{X_t\}_{t\geq 0}$ be a temporally homogeneous Markov process with values in a measurable space $(S,\mathscr{B})$ and let $V(x)\geq 0$ be a bounded measurable function on S. Suppose there exists a function $h(s)$, $(s>0)$ which tends to infinity as s goes to 0 such that

$$(1.1) \qquad E_x\left[\int_0^\infty e^{-su}\,V(X_u)\,du\right]\sim h(s), \qquad \text{as } s\to 0$$

uniformly on $\{x\mid V(x)>0\}$. Then,

Theorem B ([4]). (i) *If*

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