J. Math. Kyoto Univ. (JMKYAZ)
36-4 (1996) 809-814

Homological stability of oriented configuration spaces

By

Martin A. GUEST, Andrzej KOZLOWSKY and Kohhei YAMAGUCHI

§1. Introduction

For a connected space M, let $F(M, d)$ be space *ordered configurations* of d distinct points in, M, which is defined by

$$F(M, d) = \{(x_1, \cdots, x_d)\} \in M^d: x_i \neq x_j \text{ if } i \neq j\}.$$

Let Σ_d be the symmetric group of d letters $\{1, 2, \cdots, d\}$. Σ_d acts on $F(M, d)$ freely in the usual manner. The orbit space

$$C_d(M) = F(M, d)/\Sigma_d$$

is called the space of *configuratons* of d distinct points in M. In this paper we shall assume that M is an open manifold, i.e. each component is non-compact and without boundary. Adding a point near one of the ends of M gives (up to homotopy) a stabilization map

$$j_d: C_d(M) \rightarrow C_{d+1}(M).$$

The following is well-known:

Theorem 0 (F. Cohen [6], G. Segal [11]). *If M is an open manifold, then the stabilization map j_d: $C_d(M) \rightarrow C_{d+1}(M)$ is a homology equivalence up to dimension* [d/2].

(We shall call a map f: $X \rightarrow Y$ a *homology equivalence up to dimension m if the induced homomrphism*

$$f_*: H_i(X, \mathbf{Z}) \rightarrow H_i(Y, \mathbf{Z})$$

is bijective when $i < m$ and surjective when $i = m$.)

Remarks. Various special cases of this result were known earlier. For example.
(1) Let $M = R^q$ $(q > 2)$. Then $\lim_{q\to\infty} C_d(\mathbf{R}^q) = K(\Sigma_d, 1)$. The homology stabilization of this space follows from work of Nakaoka ([10]). We can also show this using theorem 0.

Communicated by Prof. A. Kono, January 29, 1996