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On the decomposition numbers of the Hecke algebra of *G(m, 1, n)*

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1. Introduction

It is recently found that many complex reflection groups have deformation of group algebras [A1], [AK], [BM]. For the group G $(m, 1, n)$ in the Shephard-Todd notation [C], [ST], the Hecke algebra $\mathcal{H}_A$ is the algebra over the polynomial ring $A=\boldsymbol{Z}[\boldsymbol{v}_1, \cdots, \boldsymbol{v}_m, \boldsymbol{q}, \boldsymbol{q}^{-1}]$ defined by generators $a_1, \cdots, a_n$ and relations

$$(a_1-\boldsymbol{v}_1)\cdots(a_1-\boldsymbol{v}_m)=0,\ (a_i-\boldsymbol{q})(a_i+\boldsymbol{q}^{-1})=0\,(2\leq i\leq n)$$
$$a_1a_2a_1a_2=a_2a_1a_2a_1,\quad a_ia_j=a_ja_i\,(j\geq i+2)$$
$$a_ia_{i+1}a_i=a_{i+1}a_ia_{i+1}\,(2\leq i\leq n-1)$$

This algebra is known to be A-free. If we specialize it to $\boldsymbol{v}_i=v_i$, $\boldsymbol{q}=q$, where $v_i\in \boldsymbol{C}$, $q\in \boldsymbol{C}^{\times}$, this algebra is denoted by $\mathcal{H}_{C}$.

We note here that the study of this algebra over a ring of integers is conjecturely related to the modular representation theory for the block algebras of the general linear group [BM].

One of the building blocks for the modular representation theory of $\mathcal{H}_C$ is the case that $v_1,\cdots, v_m$ are powers of $q^2\neq 1$, and we consider this case in this paper.

Let u_n be the Grothendieck group of the category of $\mathcal{H}_C$-modules. We set $u=\oplus u_n$. The purpose of this paper is to show that the graded dual of u is a highest weight module of $\mathfrak{g}(A_\infty)$ (resp. $\mathfrak{g}(A^{(1)}_{r-1})$) if q^2 is not root of unity (resp. a primitive r-th root of unity), and the dual basis of irreducible modules coincides with canonical basis. The proof heavily depends on Lusztig's theory of affine Hecke algebras and quantum groups, and Ginzburg's theory of affine Hecke algebras.

For $m=1$, our result verifies a conjecture of [LLT]. Hence their conjectural algorithm actually computes the decomposition numbers of the Hecke algebra of type A. We note here that there is an announcement of Grojnowski [Gr] on the decomposition numbers of the Hecke algebra of type A, but what we see here is that we can avoid the result at roots of unity to compute the de-

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