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Adjoint action on homology mod 2 of E_8 on its loop space

By

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1. Introduction

Assume G is a compact, connected, simply connected Lie group. The space of free loops on G is called $LG(G)$ the free loop group of G, whose multiplication is defined as

$$\varphi \cdot \phi(t) = \varphi(t) \cdot \phi(t).$$

Let ΩG be the space of based loops on G, whose base point is the unit e. Then $LG(G)$ has ΩG as its normal subgroup and

$$LG(G) / \Omega G \cong G.$$

Identifying elements of G with constant maps from S^1 to G, $LG(G)$ is equal to the semidirect product of G and ΩG. Thus the mod p homology of $LG(G)$ is determined by the mod p homology of G and ΩG and the algebra structure of $\mathbf{H}_*(LG(G) ; \mathbf{Z}/p\mathbf{Z})$ depends on $\mathbf{H}_*(\mathrm{ad} ; \mathbf{Z}/p\mathbf{Z})$ where

$$\mathrm{ad} : G \times \Omega G \rightarrow \Omega G$$

is the adjoint map.

In [4] some properties of ad_* are studied and it is showed that $\mathrm{H}_*(\mathrm{ad} ; \mathbf{Z}/p\mathbf{Z})$ is equal to $\mathbf{H}_*(p_2 ; \mathbf{Z}/p\mathbf{Z})$ where p_2 is the second projection if and only if $\mathbf{H}^*(G ; \mathbf{Z})$ is p-torsin free. For an exceptional Lie group G, $\mathbf{H}^*(G ; \mathbf{Z})$ has p-torsion when

$$\begin{aligned} G &= G_2, F_4, E_6, E_7, E_8 && \text{for } p = 2, \\ G &= F_4, E_6, E_7, E_8 && \text{for } p = 3, \\ G &= E_8 && \text{for } p = 5. \end{aligned}$$

The case where $p=2$ and $G \neq E_8$ is discussed in [6] and the case of $p=3, 5$ is studied in [8, 7] respectively. In this paper we offer the result of the remained case, $(G, p) = (E_8, 2)$. The result is showed in Theorem 4. 1.

This paper is organixed as follows. In §2 we refer to the result of the algebra structure of $\mathbf{H}^*(G ; \mathbf{Z}/2\mathbf{Z})$ and $\mathbf{H}_*(\Omega G ; \mathbf{Z}/2\mathbf{Z})$ and the Hopf algebra

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