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A duality theorem in Hopf algebras and its application to Morava K-theory of $B\mathbf{Z}/p^r$

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0. Introduction

Let $E^*(\ )$ be a complex orientable theory. Then choosing an orientation class $x \in E^2(BS^1)$, we have an isomorphism $E^*(BS^1) \cong E_*[[x]]$, where E_* is the coefficient ring. Let N be a natural number and let $[N]x = x +_F \cdots +_F x$ (N times) be the N-sequence, where $x +_F y = F(x, y)$ is the formal group law of the theory E^*. Note that $[N]x$ is the Euler class of the standard S^1-bundle

$$S^1 \to B\mathbf{Z}/N \to BS^1$$

Therefore if $[N]x$ is not a zero-divisor, from the Gysin sequence it follows that

$$E^*(B\mathbf{Z}/N) \cong E_*[[x]]/([N]x).$$

Suppose that $E^*(B\mathbf{Z}/N)$ is a finitely generated free E_*-module. Then

$$E^*(B\mathbf{Z}/N \times B\mathbf{Z}/N) \cong E^*(B\mathbf{Z}/N) \otimes_{E_*} E^*(B\mathbf{Z}/N)$$

and the product map $m : B\mathbf{Z}/N \times B\mathbf{Z}/N \to B\mathbf{Z}/N$ imduces a ring homomorphism

$$m^* : E^*(B\mathbf{Z}/N) \to E^*(B\mathbf{Z}/N) \otimes_{E_*} E^*(B\mathbf{Z}/N).$$

Thus $E^*(B\mathbf{Z}/N)$ is a bicommutative Hopf algebra over E_* and so is its dual

$$\mathrm{hom}_{E_*}(E^*(B\mathbf{Z}/N), E_*).$$

In this paper we shall study a duality between the algebraic groups of such Hopf algebras and their duals. For typical application we consider the p-adic Morava $K(n)$-theory. Let $\overline{K(n)}^*(\ )$ be the p-adic Morava $K(n)$-theory of period 2 so that the coefficient ring is

$$\overline{K(n)}_* = \mathbf{Z}_p[v_n, v_n^{-1}, t, t^{-1}]/(t^{p^n-1} - v_n)$$

where $\deg t = 2$ and $\mathbf{Z}_p$ is the ring of p-adic integers. For a $\mathbf{Z}_p$-algebra R we define

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