J. Math. Kyoto Univ. (JMKYAZ)
36-4 (1996) 687-695

Bogomolov conjecture for curves of genus 2 over function fields

By

Atsushi MORIWAKI

1. Introduction

Let k be an algebraically closed field, X a smooth projective surface over k, Y a smooth projective curve over k, and $f: X \to Y$ a generically smooth semi-stable curve of genus $g \geq 2$ over Y. Let K be the function field of Y, $\overline{K}$ the algebraic closure of K, and C the generic fiber of f. Let $j: C(\overline{K}) \to \mathrm{Jac}(C)(\overline{K})$ be a morphism given by $j(x) = (2g-2)x - \omega_c$ and $\| \ \|_{NT}$ the semi-norm arising from the Néron-Tate height pairing on $\mathrm{Jac}(C)(\overline{K})$. We set

$$B_C(P; r) = \{x \in C(\overline{K}) \mid \|j(x) - P\|_{NT} \leq r\}$$

for $P \in \mathrm{Jac}(C)(\overline{K})$ and $r \geq 0$, and

$$r_C(P) = \begin{cases} -\infty & \text{if } \#(B_C(P; 0)) = \infty \\ \sup\{r \geq 0 \mid \#(B_C(P; r)) < \infty\} & \text{otherwise.} \end{cases}$$

Bogomolov conjecture claims that, if f is non-isotrivial, then $r_C(P)$ is positive for all $P \in \mathrm{Jac}(C)(\overline{K})$. Even to say that $r_C(P) \geq 0$ for all $P \in \mathrm{Jac}(C)(\overline{K})$ is non-trivial because it contains Manin-Mumford conjecture, which was proved by Raynaud. Further, it is well known that the above conjecture is equivalent to say the following.

Conjecture 1.1 (Bogomolov conjecture). If f is non-isotrivial, then

$$\inf_{P \in \mathrm{Jac}(C)(\overline{K})} r_C(P) > 0.$$

Moreover, we can think the following effective version of Conjecture 1.1.

Conjecture 1.2 (Effective Bogomolov conjecture). In Conjecture 1.1, there is an effectively calculated positive number r_0 with

$$\inf_{P \in \mathrm{Jac}(C)(\overline{K})} r_C(P) \geq r_0.$$

Received November 20, 1995
We would like to express our thanks to Prof. Liu for his nice remarks.