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Homology ring mod 2 of free loop groups of exceptional Lie groups

By

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1. Introduction

Assume G is a compact, connected, simply connected Lie group. The space of free loops on G is called $LG(G)$ the free loop group of G, whose multiplication is defined as

$$\varphi \cdot \psi(t) = \varphi(t) \cdot \psi(t).$$

Let ΩG be the space of based loops on G, whose base point is the unit e. Then $LG(G)$ has ΩG as its normal subgroup and

$$LG(G)/\Omega G \cong G.$$

Identifying elements of G with constant maps from S^1 to G, $LG(G)$ is equal to the semidirect product of G and ΩG. Thus the homology of $LG(G)$ is determined by the homology of G and ΩG and the algebra structure of $H_*(LG(G) ; \boldsymbol{Z}/2\boldsymbol{Z})$ depends on $H_*(\mathrm{ad} ; \boldsymbol{Z}/2\boldsymbol{Z})$ where

$$\mathrm{ad} : G \times \Omega G \to \Omega G$$

is the adjoint map.

The purpose of this paper is to determine $H_*(\mathrm{ad} ; \boldsymbol{Z}/2\boldsymbol{Z})$ for the exceptional Lie goups $G = G_2$, F_4, E_6 and E_7. And at the same time, using the Hopf algebra structures of $H_*(\Omega E_6 ; \boldsymbol{Z}/2\boldsymbol{Z})$ and $H_*(\Omega E_7 ; \boldsymbol{Z}/2\boldsymbol{Z})$, we could determine the $\mathcal{A}_2^*$ module structure of $H_*(\Omega G ; \boldsymbol{Z}/2\boldsymbol{Z})$. Moreover some mistakes was detected in the result about Hopf structure of $H_*(\Omega E_6 ; \boldsymbol{Z}/2\boldsymbol{Z})$ of [5] and we offer the modified result. The main result is showed in Theorem 4.6, 4.9 and 5.11.

This paper is organized as follows. In §2 we refer to the result of the algebra structure of $H^*(G ; \boldsymbol{Z}/2\boldsymbol{Z})$ and $H^*(\Omega G ; \boldsymbol{Z}/2\boldsymbol{Z})$. And in §3 we introduce the adjoint action and observe its property and in §4, §5 the induced homomorphism from adjoint action of G_2, F_4, E_6 and E_7 is determined. Finally in §6 we give the method to compute the Pontrjagin ring of $LG(G)$ and show the case of G_2.

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