J. Math. Kyoto Univ. (JMKYAZ)
36-4 (1996) 659-668

Rectilinear slit conformal mappings

Dedicated to Professor Yukio Kusunoki on his 70th birthday

By

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1. Introduction

Let G be a region in the extended complex plane and $\widehat{G}$ be the Kerékjártó-Stoïlow compactification. Take a real-valued function $\widehat{\varphi}$ on $\widehat{G}-G$ which we call an angle assignment. Our purpose is to give a rectilinear slit conformal mapping on G such that it maps each boundary component p to a slit which lies on a line of inclination $\widehat{\varphi}(p)$ to the positive real axis, where a slit may be a point. This rectilinear slit mapping is said to achieve the angle assignment $\widehat{\varphi}$. Koebe [KP] showed the following. In the case G is a finitely connected domain, there exists a unique rectilinear slit mapping with a normalization which achieves an arbitrary given angle assignment. On the other hand, in the case G has a countable number of boundary components, there exist angle assignments which are not achieved (cf. [W]). If G has an uncountable number of boundary components, even parallel slit mapping with a normalization is not always unique as a region whose boundary consists of parallel slits of positive measure. We follow the suggestion of B. Rodin [ABB] and assume that the angle assignment $\widehat{\varphi}$ is continuous. In this paper, we assume further additional conditions about $\widehat{\varphi}$ and by Shiba's theorem [S] argue the uniqueness and existence of rectilinear slit mapping. Since the normalized rectilinear slit mapping with extremal crossing module, which F. Weening [W] gave, has boundary behavior as in Shiba's theorem, we can prove the uniqueness.

We are grateful to Dr. Frederick Weening.

2. Notation and Preliminaries

Let $\Lambda=\Lambda(G)$ be the real Hilbert space of square integrable complex differentials whose inner product is given by

$$\langle\omega,\ \sigma\rangle = \text{real part of} \iint_G \omega\wedge{}^*\bar{\sigma} = \Re(\omega,\ \sigma),$$

*Research supported by a Fellowship of the Education Ministry of Japan.
†I want to thank the University of Cincinnati for its hospitality during my stay.
Communicated by Prof. K. Ueno, November 6, 1995