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On H-spaces and exceptional Lie groups

By

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0. Introduction

An H-space is a space which admits a continuous product with unit. F. Borel [1] showed its fundamental group is restricted by the rational cohomology algebra under a certain associativity condition. In particular, if an H-space X satisfies $\mathrm{H}^*(X; \boldsymbol{Q}) \cong \mathrm{H}^*(G: \boldsymbol{Q})$ as an algebra where G is an exceptional Lie group, then $\pi_1(X)$ is a subgroup of the group in the following table.

$G = G_2$	$\pi_1(X) \subset \boldsymbol{Z}/2$
F_4	$\boldsymbol{Z}/8 \times \boldsymbol{Z}/8$
E_6	$\boldsymbol{Z}/8 \times \boldsymbol{Z}/8 \times \boldsymbol{Z}/3 \times \boldsymbol{Z}/5$
E_7	$\boldsymbol{Z}/8 \times \boldsymbol{Z}/8$
E_8	$\boldsymbol{Z}/8 \times \boldsymbol{Z}/8$

As for the mod 2 cohomology, J.Lin showed

Theorem 1 ([4]) *Let X be a 1-connected H-space such that $\mathrm{H}_*(X ; \boldsymbol{F}_2)$ is finite and associative. If $\mathrm{H}^*(X ; \boldsymbol{Q}) \cong \mathrm{H}^*(G ; \boldsymbol{Q})$ as an algebra for an exceptional Lie group G, then $\mathrm{H}^*(X ; \boldsymbol{F}_2) \cong \mathrm{H}^*(G ; \boldsymbol{F}_2)$ as an algebra over the mod 2 Steenrod algebra.*

By adding Serre spectral sequence arguments we can refine these. The purpose of this paper is to prove the following theorem.

Theorem 2 *Let X be a connected homotopy associative H-space such that $\mathrm{H}_*(X ; \boldsymbol{F}_2)$ is finite. Assume that $\mathrm{H}^*(X ; \boldsymbol{Q}) \cong \mathrm{H}^*(G ; \boldsymbol{Q})$ as an algebra, where G is an exceptional Lie group. Then $\pi_1(X)$ and $\mathrm{H}^*(X ; \boldsymbol{F}_2)$* are as follows.

$G = G_2, F_4, E_8$ $\quad \begin{cases} \pi_1(X) = 0, \\ \mathrm{H}^*(X ; \boldsymbol{F}_2) \cong \mathrm{H}^*(G ; \boldsymbol{F}_2) \end{cases}$

$G = E_6$ $\quad \begin{cases} \pi_1(X) \subset \boldsymbol{Z}/3 \times \boldsymbol{Z}/5, \\ \mathrm{H}^*(X ; \boldsymbol{F}_2) \cong \mathrm{H}^*(E_6 ; \boldsymbol{F}_2) \end{cases}$

$G = E_7$ $\quad \begin{cases} \pi_1(X) = 0, \\ \mathrm{H}^*(X ; \boldsymbol{F}_2) \cong \mathrm{H}^*(E_7 ; \boldsymbol{F}_2) \end{cases}$ or $\begin{cases} \pi_1(X) = \boldsymbol{Z}/2, \\ \mathrm{H}^*(X ; \boldsymbol{F}_2) \cong \mathrm{H}^*(\mathrm{Ad}\ (E_7) ; \boldsymbol{F}_2) \end{cases}$

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