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Limit theorems for local times of fractional Brownian motions and some other self-similar processes

By

Narn-Rueih SHIEH

1. Introduction and Main Results

A real-valued stochastic process $X(t)$, $t \geq 0$, is called *self-similar with exponent H* (H-ss for brevity) if $X(ct) \stackrel{d}{=} c^H X(t)$ for each $c > 0$. It is called *of stationary increments* (si for brevity) if $X(t+b) - X(b) \stackrel{d}{=} X(t) - X(0)$ for each $b > 0$. The notation $\stackrel{d}{=}$ in the above means the finite-dimensional equivalence of two processes. In this paper, we consider the exponent $H : 0 < H < 1$, and thus $X(0) = 0$. One may refer to Maejima (1989) and Vervaat (1987) for intensive surveys on self-similar proccesses. We also assume that X is of continuous paths or of cadlag paths (cadlag = right-continuous with left-limits everywhere). Under a main assumption of "approximately independent increments", as we shall see in §2, almost every path $X(\cdot, \omega)$ has regular local times $L(t, x, \omega)$, $t \geq 0$ and $x \in \mathbf{R}$. We consider the following functionals :

$$
\begin{aligned}
F_0(t, \omega) &= L(t, 0, \omega), \\
F_1(t, \omega) &= \sup_x L(t, x, \omega), \\
F(\mu, t, \omega) &= \int_{-\infty}^{\infty} L(t, x, \omega)\, \mu(dx), \\
F(f, t, \omega) &= \int_{-\infty}^{\infty} L(t, x, \omega)\, f(x)\, dx = \int_0^t f(X(s, \omega))\, ds,
\end{aligned}
$$

where μ is a finite Borel measure on the line and f is a Lebesgue integrable function on the line. The above functionals, regarded as continuous-paths processes in t, have their own self-similarity. In this paper, we prove some limit theorems for the rescalings of these functionals. Let $\stackrel{w}{\rightarrow}$ denote the weak convergence in the law of the space of continuous functions, we have

Theorem 1. *As* $\lambda \rightarrow \infty$,

$$
\left\{ \frac{F(\mu, \lambda t)}{\lambda^{1-H}} \right\}_{t \geq 0} \stackrel{w}{\rightarrow} \left\{ \mu(\mathbf{R}) \cdot L(t, 0) \right\}_{t \geq 0},
$$

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