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Adjoint actions on the modulo 5 homology groups of E_8 and ΩE_8

By

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1. Introduction

Borel proved in [2] that the integral homology group of the exceptional Lie group E_8 is not 5-torsion free and

$$H(E_8;Z/5) \cong \Lambda(x_3,x_{11},x_{15},x_{23},x_{27},x_{35},x_{39},x_{47}) \otimes Z/5[x_{12}]/(x_{12}^5), \text{ with } |x_i|=i,$$

as algebra.

Araki showed the non-commutativity of the Pontrjagin ring $H_*(E_8;Z/5)$ in [1]. The whole Hopf algebra structure and the cohomology operations were determined by Kono in [6]. But it was due to the partial computation of $\mathrm{Cotor}^{H^*(E_8;Z/5)}(Z/5,Z/5)$, which was rather complicated. In [5], using secondary cohomology operations, Kane gave a general theorem to determine the Pontrjagin ring which is non-commutative and determined $H_*(E_8;Z/5)$ as a Hopf algebra over $\mathscr{A}_5$.

Also, for a compact, connected Lie group G, the free loop group of G denoted by $LG(G)$ is the space of free loops on G equiped with multiplication as

$$\phi\cdot\psi(t)=\phi(t)\cdot\psi(t),$$

and has ΩG as its normal subgroup. Thus

$$LG(G)/\Omega G \cong G,$$

and identifying elements of G with constant maps from S^1 to G, $LG(G)$ is equal to the semi-direct product of G and ΩG. This means that the homology of $LG(G)$ is determined by the homology of G and ΩG as module and the algebra structure of $H_*(LG(G);Z/p)$ depends on $H_*(\mathrm{Ad};Z/p)$ where

$$\mathrm{Ad}: G\times\Omega G \to \Omega G$$

is the adjoint map. Since the next diagram commutes where λ,λ' and μ are the multiplication maps of ΩG, $LG(G)$ and G respectively and ω is the composition

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