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On May spectral sequences

Dedicated to Professor Teiichi Kobayashi on his 60th birthday

By

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1. Introduction

In this paper, we generalize the algebraic Novikov spectral sequences. Then we define a homomorphism from the algebraic Novikov spectral sequence to the May spectral sequence and study conditions in which they are isomorphic.

In [3], we have indicated that the differentials of the generalized Adams spectral sequences can be calculated from those of the May spectral sequences. Moreover we can calculate the algebraic Novikov spectral sequences (c.f., Proposition 2.5). Hence the results of this paper can be applied to the calculation of the differentials of the generalized Adams spectral sequences. We may apply to the BP-and $E(n)$-Adams spectral sequences, and find the new elements of the stable homotopy groups of spheres and the $E(n)$-localization of spheres. These results will appear in the forthcomming papers.

Let F be a ring spectrum with unit τ^F: $S^0 \to F$, $S^0 \xrightarrow{\tau^F} F \xrightarrow{pr} \overline{F}$ the cofiber of τ^F and $\overline{F}^s = \overline{F} \wedge \cdots \wedge \overline{F}$ the s-fold smash product of $\overline{F}$. For any CW spectrum X, we have exact sequences

$$\cdots \xrightarrow{\partial^F} \pi_u(S^0 \wedge \overline{F}^s \wedge X) \xrightarrow{(\tau^F \wedge 1)*} \pi(F \wedge \overline{F}^s \wedge X) \xrightarrow{(pr \wedge 1)*} \pi(\overline{F} \wedge \overline{F}^s \wedge X) \xrightarrow{\partial^F} \pi_{u-1}(\overline{F}^s \wedge X) \to \cdots$$

and a filtration

$$\pi_u(X) \xleftarrow{\partial^F} \pi_{u+1}(\overline{F} \wedge X) \xleftarrow{\partial^F} \cdots \xleftarrow{\partial^F} \pi_{u+s}(\overline{F}^s \wedge X) \xleftarrow{\partial^F} \cdots.$$

The F-Adams spectral sequence $\{{}_FE_r^{s,u}(X),\ d_r^F\}$ is the spectral sequence induced from the exact couple consisting of the above long exact sequence. We have

$${}_FE_1^{s,u}(X) = F_u(\overline{F}^s \wedge X),\ d_1^F = (\tau^F \wedge 1)_* \circ (pr \wedge 1)_* \text{ and } {}_FE_2^{s,u}(X) = H^s({}_FE_1^{*,u}(X);\ d_1^F)$$

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