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A note on pluricanonical maps for varieties of dimension 4 and 5

By

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1. Introduction

Let X be a nonsingular projective variety of general type with dimension n defined over $\mathbf{C}$. The behavior of its pluricanonical map $\Phi_{|mK_X|}$ is of special interest to the classification theory. For $n \geq 3$, it remains open whether there is an absolute function $m(n)$ such that $\Phi_{|mK_X|}$ is birational for $m \geq m(n)$. The simplest case to this problem is when X be a nonsingular minimal model. For $n \geq 4$, T. Matsusaka first proved the existence of $m(n)$; K. Maehara presented a function $m(n)$; T. Ando ([1]) got $m(4)=16$ and $m(5)=29$.

With I. Reider's results ([6]) and by improving T. Ando's method, we get the following effective result.

Theorem. *Let X be a nonsingular projective variety of dimension $n \geq 4$ with nef and big canonical divisor K_X. Then there is a function $m(n)$ such that $\Phi_{|mK_X|}$ is birational for $m \geq m(n)$, where $m(4) \leq 12$ and $m(5) \leq 18$.*

Throughout this note, most of our notations and terminologies are standard except the following which we are in favour of:

$:=$ —— definition;

$\sim_{lin}$ —— linear equivalence;

$\sim_{num}$ —— numerical equivalence.

2. The main theorem

We begin by introducing I. Reider's result at first.

Lemma 2.1 *(Corollary 2 of [6]). Let S be an algebraic surface, L a nef and big divisor on S. Suppose $L^2 \geq 10$ and the rational map ϕ defined by $|L+K_S|$ is not birational, then S contains a base point free pencil E' with $L \cdot E'=1$ or $L \cdot E'=2$.*

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