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A natural horseshoe-breaking family which has a period doubling bifurcation as the first bifurcation

By

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1. Introduction

Let $F: M \to M$ be a diffeomorphism on a manifold M. We say that F is C^r-*structurally stable* if there is a homeomorphism $h: M \to M$ which satisfies $h^{-1} \circ G \circ h = F$ for any diffeomorphism G near F in C^r-topology.

F is called *hyperbolic* on an F-invariant compact set Λ if there exist a splitting of the tangent bundle $TM|_\Lambda = E^u \oplus E^s$ and constants $C>0$ and $\lambda>1$ which satisfy that

$$\| DF^n(v_u) \| \geq C\lambda^n \|v_u\|, \ \| DF^{-n}(v_s) \| \geq C\lambda^n \|v_s\|$$

for any $n \in \mathbf{N}$ and any $v_u \in E^u$, $v_s \in E^s$. We call F *a hyperbolic diffeomorphism* if the non-wandering set of F is hyperbolic.

The hyperbolicity and the stability are very important concepts in the research of dynamical systems and they are mutually related. For example, it is known that F is C^1-structurally stable, if and only if F is hyperbolic and F has some additional conditions (*e.g.* [7] and [3]).

Smale's horseshoe [8] is a typical dynamical system which has the stability and the hyperbolicity, A horseshoe is defined as a planer diffeomorphism which maps D to $F(D)$ as in Figure 1. and which is hyperbolic on $\Lambda = \bigcap_{n \in \mathbf{Z}} F^n(D)$. We call horseshoe diffeomorphism *an affine horseshoe* if it maps the rectangles A and B to A' and B' affinely.

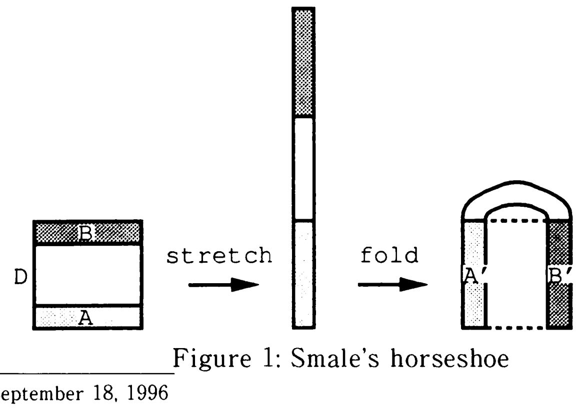

Figure 1: Smale's horseshoe

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