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The transverse structure of Lie flows of codimension 3

By

Blas HERRERA and Agustí REVENTÓS

1. Introduction

This paper deals with the problem of the realization of a given Lie algebra as transverse algebra to a Lie foliation on a compact manifold.

Lie foliations have been studied by several authors ([E.H.S], [E.N], [F], [H.M], [M], [Ma], etc.). The importance of this study was increased by the fact that they arise naturally in Molino's classification of Riemannian foliations [M].

To each Lie foliation are associated two Lie algebras, the Lie algebra $\mathcal{G}$ of the Lie group on which the foliation is modeled and the structural Lie algebra $\mathcal{H}$. The latter algebra is the Lie algebra of the Lie foliation $\mathcal{F}$ restricted to the closure of any one of its leaves. In particular, it is a subalgebra of $\mathcal{G}$. We remark that although $\mathcal{H}$ is canonically associated to $\mathcal{F}$, $\mathcal{G}$ is not.

Thus two interesting problems are naturally posed: the *realization problem* and the *change problem*.

The *realization problem* is to know which pairs of Lie algebras $(\mathcal{G}, \mathcal{H})$, with $\mathcal{H}$ subalgebra of $\mathcal{G}$, can arise as transverse and structural Lie algebras, respectively, of a Lie foliation $\mathcal{F}$ on a compact oriented manifold M.

This problem is closely related to the following Haefliger's problem [Ha]: given a Lie subgroup Γ of a Lie group G, is there a Lie G-foliation on a compact manifold M with holonomy group Γ? E. Ghys [Gh] and G. Meigniez [Mg] also studied this problem and they gave necessary conditions for a pair (G, Γ) to be realizable.

Our formulation of the realization problem is a little different: We shall say that the pair $(\mathcal{G}, q)$ is *realizable* if there is a compact oriented manifold endowed with a Lie foliation transversely modeled on $\mathcal{G}$ and with structural Lie algebra of dimension q. We also say that $\mathcal{G}$ is realizable as *transverse* to a Lie foliation.

This formulation of the *realization problem* has been considered in [Ll], [H], [G, R] and [H.Ll.R] making a very detailed study of Lie flows of codimension 3 (cf. §8). But a complete classification was not obtained because of the following open question:

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