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Remarks on L^2-wellposed Cauchy problem for some dispersive equations

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1. Introduction and results

We consider the Cauchy problem with data on line $t=0$ for the following operator A defind by

$$Au(t,x)=\frac{\partial}{\partial t}u(t,x)+\frac{\partial^3}{\partial x^3}u(t,x)+a(x)\frac{\partial}{\partial x}u(t,x)+b(x)u(t,x) \qquad (1.1)$$

with the complex-valued coeffcients $a(x)$ and $b(x)$ belonging to the space B^∞ consisting of all bounded smooth functions whose derivative of any order is also bounded on real line $\mathbf{R}$.

If the coefficients $a(x)$ and $b(x)$ are constant, we see by Fourier transformation that, when the imaginary part of the coefficent $a(x)$ is not zero, the Cauchy problem for A is not L^2-wellposed.

This implies that the Cauchy problem for A is not always L^2-wellposed. Indeed we see by the construction of asymptotic solutions that the following condition on the imaginary part of the coefficent $a(x)$, which is denoted by $a_I(x)$: there exists a constant K such that we have for any x and $y\in\mathbf{R}$

$$\left|\int_x^y a_I(s)\,ds\right|\le K|x-y|^{\frac{1}{2}} \qquad (\mathrm{N})$$

is necessary for L^2-wellosedness.

Our main interest is the sufficiency of (N). We show in this paper that the condition (N) implies L^2-wellposedness.

Now we formulate the Cauchy problem. Let T be some given positive number. For given functions $g(x)$ and $f(t,x)$ find a solution $u(t,x)$ satisfying

$$\begin{cases} Au(t,x)=f(t,x) & \text{on } [0,T]\times\mathbf{R} \\ u(0,x)=g(x) & \text{on } \mathbf{R} \end{cases} \qquad (\mathrm{C})$$

Let X be a subspace of the space of temperate distributions on $\mathbf{R}$. We say that the above problem (C) is X-wellposed if for any $g(x)\in X$ and $f(t,x)$

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