

## Recurrence and conservativeness of symmetric diffusion processes by Girsanov transformations

Dedicated to Professor Hiroshi Kunita on the occasion of his sixtieth birthday

By

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### 1. Introduction

Recurrence and conservativeness of symmetric Markov processes have been studied by several people. Firstly K. Ichihara [13], M. Fukushima [8] gave a recurrence criterion for symmetric diffusion processes on  $\mathbf{R}^d$ . K. Ichihara [16] also gave a conservativeness test for such processes. Y. Oshima [24] extended their criteria and gave the general criteria of recurrence and conservativeness for symmetric Markov processes. M. Takeda [32] also gave the conservativeness test for diffusion processes on  $\mathbf{R}^d$ , which is sharper than [16], [24], by using the Lyons-Zheng decomposition. On the other hand, K. Ichihara [14], [15], M. P. Gaffney [10], S. Y. Cheng and S. T. Yau [2], A. A. Grigor'yan [11], [12], and M. Takeda [33] gave the criteria for Brownian motions on Riemannian manifolds. K. Th. Sturm [30] extended their works for strong local regular Dirichlet forms by using the Carathéodory (intrinsic) metric. He assumed the relative compactness of balls by this metric (see also [31]). H. Ôkura [23] also gave the recurrence criteria for regular Dirichlet forms by using the capacity inequality. He assumed the local integrability of metric by jumping measure with the relative compactness of balls. H. Kaneko [17], [18] extended the results of [23], [30]. He used a class of exhaustion functions instead of the Carathéodory metric. In the case of regular Dirichlet forms on locally compact state space, M. Takeda [35] gave the conservativeness test for symmetric diffusion processes transformed by supermartingale multiplicative functionals by using the Carathéodory metric with the relative compactness of balls. Also Y. Oshima [25], Y. Oshima and K. Th. Sturm [26] gave a criterion of conservativeness for the time-dependent Dirichlet forms. In this paper, we will give the recurrence and conservativeness criteria for symmetric diffusion processes on a separable

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