J. Math. Kyoto Univ. (JMKYAZ)
37-4 (1998) 701-715

Elliptic cohomology of classifying spaces of cyclic groups and higher level modular forms

By

SHUMA MORIMOTO and GORO NISHIDA

0. Introduction

The subject of elliptic cohomology $Ell^*(-)$ defined by P. S. Landweber, D. C. Ravenel and R. E. Stong (see [14]) is one of the most important subjects in algebraic topology. They used the elliptic curve defined by the Jacobi quartic $y^2=1-2\delta x^2+\varepsilon x^4$ in projective 3-space and the associated formal group law (so called the Euler formal group law):

$$F(x, y)=\frac{x\sqrt{R(y)}+y\sqrt{R(x)}}{1-\varepsilon x^2y^2},$$

where $R(x) = 1 - 2\delta x^2 + \varepsilon x^4$. The coefficient ring Ell_* is identified with $\mathbf{Z}[\frac{1}{2}][\delta, \varepsilon, \Delta^{-1}]$ the ring of meromorphic modular forms on Γ_θ over $\mathbf{Z}[\frac{1}{2}]$.

Later A. Baker [1] has defined elliptic cohomology based on the modular forms on $SL_2(\mathbf{Z})$ over $\mathbf{Z}[\frac{1}{6}]$ and the elliptic cohomologies of higher level have been defined by J-L. Brylinski (cf. [3]). A. Baker [2] has shown that given a prime $p>3$, the supersingular reduction of $Ell^*(-)$ at p, namely reduction with respect to the ideal I_2 generated by p and v_1, is essentially isomorphic to the Morava $K(2)$-theory.

On the other hand, T. Torii [17] has shown the following. Let $B\mathbf{Z}/(p^n)$ be the classifying space of the cyclic group $\mathbf{Z}/(p^n)$ for a prime p and $\overline{K(r)}^*(-)$ be the p-adic Morava K-theory, then the ring $\overline{K(r)}^*(B\mathbf{Z}/(p^n))$ is described as the totally ramified extension of $\overline{K(r)}_* \cong \mathbf{Z}_p[\zeta_{p^n-1}]$, obtained by adding the roots of the equation $[p^n](x) = 0$ for the p^n-sequence of the Lubin-Tate formal group law of height r. Where we denote by ζ_l a primitive l-th root of unity.

From now on, we assume that p is an odd prime. By the above result, we may expect that the elliptic cohomology $Ell^*(B\mathbf{Z}/(p^n))$ of $B\mathbf{Z}/(p^n)$ can be described by level $2p^m$ modular forms for $0 \leq m \leq n$. The purpose of this paper is to show that this is true after certain completion of Ell_*. We shall study the level 2 elliptic cohomology. Now the main result is stated as follows.

Received November 11, 1996