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Absence of diffusion near the bottom of the spectrum for a random Schrödinger operator on $L^2(\boldsymbol{R}^3)$

By

Yuji NOMURA

1. Introduction

Let $(\Omega, F, \boldsymbol{P})$ be a probability space whose precise definition will be given later. For each $\omega \in \Omega$, we consider Anderson type random Schrödinger operator on $\boldsymbol{L}^2(\boldsymbol{R}^3)$:

$$(1.1)\qquad \begin{cases} H_\omega = -\Delta + V_\omega(x), \\ V_\omega(x) = \sum_{i\in \boldsymbol{Z}^3} q_i(\omega) f(x-i) \end{cases}$$

where $\Delta = \sum_{j=1}^{3} \frac{\partial^2}{\partial x_j^2}$. $\{q_i\}_{i\in\boldsymbol{Z}^3}$ satisty

(H.1) $\{q_i\}_{i\in\boldsymbol{Z}^3}$ are real-valued independent identically distributed random variables on $(\Omega, F, \boldsymbol{P})$ with uniform distribution on $[0, 1]$.

We suppose the following conditions:

(H.2) There exist two positive numbers η_0 and η_1 such that $\eta_0 \leq f(x) \leq \eta_1$ for $x \in [0, 1)^3$,

(H.3) $x \notin [0, 1)^3 \Rightarrow f(x) = 0$.

H_ω is considered to be the operator corresponding to the Hamiltonian of the electron in random metalic media. Let $\sigma(H_\omega)$ denote the spectrum of H_ω. Then the following is a known fact.

Proposition 1.1. (Kirsch and Martinelli).

$$\sigma(H_\omega) = [0, \infty) \ a.s.$$

For $E > 0$, we shall mean by g_E an arbitrary real-valued function which satisfies the following condition:

(A) $g_E \in C_0^\infty(\boldsymbol{R})$ and supp $g_E \subset (0, E)$,

where $C_0^\infty(O) = \{f \in C^\infty(O) \mid \mathrm{supp} f \subset O\}$ for an open set $O \subset \boldsymbol{R}^n$.

In this paper we are interested in the following quantity:

$$(1.2)\qquad r_E^2(t) = \boldsymbol{E}\left[\int_{\boldsymbol{R}^3} |x|^2 |e^{-itH_\omega} g_E(H_\omega) \phi(x)|^2 dx\right]$$

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