## **Projective elements in** *K***-theory and self maps of** $\sum CP^{\infty}$

By

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## 1. Introduction and statements of results

In this paper, we will work in the homotopy category of based spaces and based maps. Given a space X, we denote the reduced K-theory by K(X) and the homology group of integral coefficients by  $H_*(X)$ . Let  $CP^{\infty}$  be the infinite dimensional complex projective space. Let  $\eta$  be the canonical line bundle over  $CP^{\infty}$  and i:  $CP^{\infty} \rightarrow BU$  be the classifying map of the virtual bundle  $\eta - 1$ . Since BU has a loop space structure which is derived from the Whitney sum of complex vector bundles, there exists a unique extension of i to the loop map  $j: \Omega \sum CP^{\infty} \rightarrow BU$ .

In this paper we investigate the following problems:

Given an element  $\alpha \in K(X)$ , when does there exist a lift  $\widehat{\alpha} \in [X, \Omega \sum CP^{\infty}]$ such that  $j_*(\widehat{\alpha}) = \alpha$ ? If  $\alpha$  has a lift, how we can construct the lift  $\widehat{\alpha}$ ?

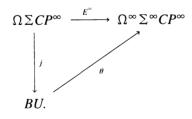
Define

$$PK(X) = \{ \alpha \in K(X) \mid \exists \ \widehat{\alpha} \in [X, \Omega \sum CP^{\infty}] \text{ such that } j_*(\widehat{\alpha}) = \alpha \}.$$

If an element  $\alpha \in K(X)$  belongs to PK(X), we call that  $\alpha$  is projective.

The significance of the above problem is as follows:

The James splitting theorem [2] implies that there exists a loop map  $\theta: BU \rightarrow \Omega^{\infty} \sum^{\infty} CP^{\infty}$  such that the following diagram commutes:



Therefore, given an element  $\alpha \in K(X)$ , we have the stable map,  $adj.(\theta(\alpha)): \Sigma^{\infty}X \rightarrow \Sigma^{\infty}CP^{\infty}$ . Using the information of K(X), we can calculate the induced homomorphism [3], [4] of  $adj.(\theta(\alpha))_*: H_*(X) \rightarrow H_*(CP^{\infty})$ . If  $\alpha$  has a lift  $\hat{\alpha}$ , then this implies that the stable map  $adj.(\theta(\alpha))$  and its induced

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