J. Math. Kyoto Univ. (JMKYAZ)
38-2 (1998), 229-240

BGG-resolution for α-stratified modules over simply-laced finite-dimensional Lie algebras

By

V. Futorny* and V. Mazorchuk

1. Introduction

This paper is a sequel of [6] where the submodule structure of α-stratified (i.e. torsion free with respect to the subalgebra corresponding to a root α) generalized Verma modules was studied. The results obtained there generalize the classical theorem of Bernstein-Gelfand-Gelfand on Verma module inclusions. The crucial role in the study is played by the generalized Weyl group W_α that acts on the space of parameters of generalized Verma modules.

Let G be a simple finite-dimensional Lie algebra over the complex numbers with a simply-laced Coxeter-Dynkin diagram (i.e. there are no multiple arrows). In the present paper for any such algebra we construct a strong BGG-resolution for α-stratified irreducible modules in the spirit of [1,10]. The non-simply-laced case is more complicated (cf. [6]). In particular, the proof of the crucial Theorem 4 is based on the fact that the diagram is simply-laced.

The structure of the paper is the following. In Section 2 we collect the notation and preliminary results. A weak generalized BGG-resolution is constructed in Section 3. Here we follow closely [1]. Section 4 contains an extension lemma for α-stratified modules which generalizes a well-known result of Rocha-Caridi for Verma modules [10]. Our proof is analogous to the one of Humphreys for Verma modules [8]. In Section 5 we study the maximal submodule of the generalized Verma module and construct a strong generalized BGG-resolution for α-stratified irreducible modules in Section 6. Finally, in Section 7 we give a character formula for certain α-stratified irreducible modules.

2. Notation and preliminary results

Let $\boldsymbol{C}$ denote the complex numbers, $\boldsymbol{Z}$ all integers, $\boldsymbol{N}$ all positive integers and $\boldsymbol{Z}_+ = \boldsymbol{N} \cup \{0\}$.

Let H be a Cartan subalgebra of G and Δ the root system of G.

Let π be a basis of Δ containing α, $\Delta_\pm = \Delta_\pm(\pi)$ the set of positive (negative) roots with respect to π. For any $S \subset \pi$ let $\Delta_\pm(S)$ be a subset generated by S (it

---

Communicated by Prof. M. Jimbo, July 15, 1996
*Regular Associate of the ICTP