J. Math. Kyoto Univ. (JMKYAZ)
38-3 (1998) 587-593

Generators of the cohomology of BV_3

By

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1. Introduction

Let $P_k = \mathbf{F}_2[x_1, \ldots, x_k]$ be a polynomial algebra over the field $\mathbf{F}_2$ generated by $x_1, \ldots, x_k$. By assigning degree 1 to each x_j, P_k is regarded as a graded algebra over the ground field $\mathbf{F}_2$. The mod 2 cohomology ring of the classifying space BV_k of the elementary abelian 2-group V_k with rank k, is isomorphic to P_k as a graded algebra. Through this isomorphism, we may regard P_k as an $\mathscr{A}$-module where $\mathscr{A}$ stands for the mod 2 Steenrod algebra.

From early days of algebraic topology, topologists have been studying this cohomology ring and by making use of this cohomology ring, topologists have been proving many theorems. But our knowledge of this cohomology ring is not deep enough. For instance, we do not know even the dimension of the vector space $QP_k^n = (\mathbf{F}_2 \otimes_{\mathscr{A}} P_k)^n$ for $k = 4$.

In this paper, we determine the upper bound for the dimension of the above vector space QP_3^n and give a set of generators in terms of monomials in Theorem 5.2 and we will see that $\dim QP_3^n \leq 21$. In his thesis [4], the author gave the proof for the linear independence of these monomials in QP_3^n. After his thesis was submitted, the lower bound for $\dim QP_3^n$ is provided by Ali, Crabb and Hubbuck in [1] and Boardman in [2] investigating homology of BV_3 instead of its cohomology. In this paper, we do not give the proof of linear independence of our generators but they actually form a minimal set of generators.

2. $\alpha(n)$ and $\beta(n)$

We begin with the definitions and elementary properties of $\alpha(n)$ and $\beta(n)$. $\alpha(n)$ is the number of 1's in the dyadic expansion of n and $\beta(n)$ is the smallest positive integer that satisfies the condition $\alpha(n+\beta(n)) \leq \beta(n)$.

We need the following properties of $\beta(n)$.

Proposition 2.1. *$\alpha(n+m) \leq m$ if and only if $\beta(n) \leq m$.*

Proof. The "*only if*" part is nothing but the definition of $\beta(n)$. The "*if*" part is shown as follows: if $\beta(n) \leq m$, then

Communicated by A. Kono, January 23, 1998