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On Morava K-theory of $B(\mathbf{Z}/p)^m$ as a representation of $m \times m$ matrices ring $M_m(\mathbf{F}_p)$

By

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1. Introduction and Main Results

Let $\overline{K(n)}^*(\ )$ be the p-adic Morava $K(n)$-theory of period 2 with the coefficient ring

$$\overline{K(n)}_* = \mathbf{Z}_p[v_n, v_n^{-1}, t, t^{-1}]/(t^{p^n-1} - v_n) = \mathbf{Z}_p[t, t^{-1}].$$

Here $\deg t = 2$ and $\mathbf{Z}_p$ is the ring of p-adic integers. For any $\mathbf{Z}_p$-algebra R we define

$$\overline{K(n)}_R^*(\ ) = \overline{K(n)}^*(\ ) \otimes R.$$

If R is a finitely generated free $\mathbf{Z}_p$-module, then $\overline{K(n)}_R^*(\ )$ will be a complex orientable cohomology theory. Throughout this paper for any local field K we denote by $\mathcal{O}_K$ its ring of integers. Then we have

$$\overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p) \cong \mathcal{O}_K[[x]]/([p]x),$$

where $[p]x$ is the p-series of the Lubin-Tate formal group law and we can choose such an orientation x, that

$$[p]x = px - x^{p^n}.$$

Thus we have that $\overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p)$ is free of rank p^n over the coefficient ring $\mathcal{O}_K$ and $\overline{K(n)}_{\mathcal{O}_K}^1(B\mathbf{Z}/p) = 0$. Hence

$$\overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p \times B\mathbf{Z}/p) \cong \overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p) \otimes_{\mathcal{O}_K} \overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p)$$

and the product map $m : \mathbf{Z}/p \times \mathbf{Z}/p \to \mathbf{Z}/p$ induces a ring homomorphism

$$(Bm)^* : \overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p) \to \overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p) \otimes_{\mathcal{O}_K} \overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p).$$

So $\overline{K(n)}_{\mathcal{O}_K}^0(B\mathbf{Z}/p)$ is a bicommutative Hopf algebra over $\mathcal{O}_K$.

Now we consider a classifying space of m-times direct product of $\mathbf{Z}/p$. It is easy to see that also $\overline{K(n)}_{\mathcal{O}_K}^0(B(\mathbf{Z}/p)^m)$ is a bicommutative Hopf algebra over $\mathcal{O}_K$. The semi-group $M_m(\mathbf{F}_p)$ of $m \times m$ matrices with entries in $\mathbf{F}_p$ acts on $(\mathbf{Z}/p)^m$

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