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Non-relativistic global limits of weak solutions of the relativistic Euler equation

By

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1. Introduction

The relativistic Euler equation for a perfect fluid in two dimensional Minkowski space-time has the form ([9], [10])

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left\{\frac{(p+\rho c^2)}{c^2}\frac{v^2}{c^2-v^2}+\rho\right\}+\frac{\partial}{\partial x}\left\{(p+\rho c^2)\frac{v}{c^2-v^2}\right\}=0,\\
&\frac{\partial}{\partial t}\left\{(p+\rho c^2)\frac{v}{c^2-v^2}\right\}+\frac{\partial}{\partial x}\left\{(p+\rho c^2)\frac{v^2}{c^2-v^2}+p\right\}=0.
\end{aligned}
\tag{1.1}
$$

Here $v=v(x,t)$ is the classical coordinate velocity, $\rho=\rho(x,t)$ is the mass-energy density of the fluid, $p=p(\rho)$ is the pressure and c is the speed of light. On the other hand, the non-relativistic Euler equation is

$$
\begin{aligned}
&\frac{\partial}{\partial t}\rho+\frac{\partial}{\partial x}(\rho v)=0,\\
&\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(\rho v^2+p)=0.
\end{aligned}
\tag{1.2}
$$

For the systems (1.1) and (1.2), the local existence theorems are known for the smooth solutions (see [4] and [5] for the full-dimensional case). Also, the global existence theorems are established for the one-dimensional isentropic motions $p=\rho^\gamma$, $\gamma>1$ ([1] and [7]). In the case of the isothermal motions $p=\sigma^2\rho$, where the sound speed σ is assumed to be the constant, the existence theorems with arbitrary initial data have been obtained both for (1.1) and (1.2), by J. Smoller and B. Temple [9] and by T. Nishida [6] respectively.

In physics, it is well-known that the classical mechanics reappears as the limit of the relativistic mechanics when $c\to\infty$, and in particular, it is easy to check that the relativistic Euler equation (1.1) reduces formally to the non-relativistic Euler equation (1.2) when $c\to\infty$. However, until now there are only local results for the limit of smooth solutions of the relativistic Euler equation ([5]). The aim of this paper is to discuss the convergence of weak solutions of (1.1) as $c\to\infty$. Since

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