J. Math. Kyoto Univ. (JMKYAZ)
38-3 (1998) 475–486

Computations of moments for discounted Brownian additive functionals

By

Sadao SATO and Marc YOR

1. Introduction

Let (B_t) be the one dimensional standard Brownian motion and (ℓ_t^x) be its local time at x. Then the discounted local time at x is defined by

$$L^x = \int_0^\infty e^{-s}\, d_s \ell_s^x. \tag{1.1}$$

And we also define the discounted time spent above x:

$$A^x = \int_0^\infty e^{-s} 1_{(B_s > x)}\, ds. \tag{1.2}$$

In [BW1], M. Baxter and D. Williams study the law of the functional $A = A^0$. In their approach, the following symmetry property is fundamental.

$$A \stackrel{law}{=} 1 - A \quad \text{under } P_0. \tag{1.3}$$

Moreover, with the help of the differentiability in x of the Laplace transform of A^x, they obtained a double recurrence formula for the moments and its asymptotic law. In [BW2], they extended their considerations to a large class of diffusion processes.

In [Y1], the author studied the joint moments of L $(= L^0)$ and A, explaining the differentiability property obtained in [BW1] as a consequence of the following formulae:

$$A^x = \int_x^\infty dy \int_0^\infty e^{-s}\, d_s \ell_s^y = \int_x^\infty dy\, L^y \tag{1.4}$$

And the symmetry property may also be extended in the joint form:

$$(L, A) \stackrel{law}{=} (L, 1 - A) \quad \text{under } P_0. \tag{1.5}$$

Then, with the help of the right and left derivatives at $x = 0$ of the joint Laplace transform of L^x and A^x, he obtained a double recurrence formula for joint moments.

Communicated by S. Watanabe, September 1, 1997