J. Math. Kyoto Univ. (JMKYAZ)
38-3 (1998) 439-451

The Neumann problem on wave propagation in a 2-D external domain with cuts

By

P. A. KRUTITSKII

1. Introduction

The theory of boundary value problems for 2-D PDE's mostly deals with connected domains bounded by closed curves. A small number of investigations are devoted to the problems outside cuts in the plane. There are almost no any results concerning the well-posedness of classical problems in domains bounded by closed curves and containing cuts. It seems, that the difficulties in the analysis of these problems proceed from the different technique of the proof of the solvability theorems for domains bounded by closed curves and for plane with cuts. It is very likely, that there is no great difference between these problems in nature. In the present note we try to overcome technical difficulties for the Helmholtz equation in an external domain with cuts and therefore to suggest approach to the analysis of similar problems.

The 2-D Neumann boundary value problem for the Helmholtz equation in a multiply connected domain bounded by closed curves is considered in monographs on mathematical physics, for instance in [1]. The review on studies of the Neumann problem for this equation in the exterior of cuts is given in [4]. The present note is attempt to join these problems together and to consider both internal and external domains containing cuts. From practical stand-point such domains have great significance, because cuts model cracks, screens or wings in physical problems. We consider the case, when the parameter in the Helmholtz equation is not an eigenvalue for corresponding single connected internal domains.

The Dirichlet problem for the propagative Helmholtz equation in a 2-D external domain with cuts has been studied in [6]. The Dirichlet problem for the dissipative Helmholtz equation in both internal and external domains with cuts has been investigated in [7]. The case of strongly dissipative Helmholtz equation has been treated in [8] under weakened conditions.

The present paper is organized as follows. Formulation of the boundary value problem and the uniqueness theorem are given in the section 2. With the help of potential theory, the problem is reduced to the boundary integral equations in the section 3. The Fredholm integral equation of the 2-nd kind is derived in

Communicated by K. Ueno, June 24, 1997