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K3 surfaces with order five automorphisms

By

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Introduction

Let T be a normal projective algebraic surface over $\mathbf{C}$ with at worst quotient singular points (= Kawamata log terminal singular points in the sense of [Ka, Ko]). T is called a *log Enriques surface* if the irregularity $h^1(T, \mathcal{O}_T) = 0$ and if a positive multiple IK_T of the canonponial Weil divisor K_T is linearly equivalent to zero. Without loss of generality, we always assume from now on that a log Enriques surface has no Du Val singular points (see the comments after [Z1, Proposition 1.3]).

The smallest integer $I > 0$ satisfying $IK_T \sim 0$ is called the (global) *index* of T. It can be proved that $I \leq 66$ (cf. [Z1]). Recently, R. Blache [Bl] has shown that $I \leq 21$. He also studied the "generalized" log Enriques surfaces where log canonical singular points are allowed.

Rational log Enriques surfaces T can be regarded as degenerations of K3 or Enriques surfaces, which in turn played important roles in Enriques-Kodaira's classification theory for surfaces. In [A], A. Alexeev [A] has proved the boundedness of families of these T. In 3-dimensional case, the base surfaces W of elliptically fibred Calabi-Yau threefolds $\Phi_{|D|} : X \to W$ with $D.c_2(X) = 0$ are rational log Enriques surfaces (cf. [O1–O4]).

Let T be a log Enriques surface of index I. The Galois $\mathbf{Z}/I\mathbf{Z}$-cover

$$\pi : Y := \mathrm{Spec}_{\mathcal{O}_T} \bigoplus_{i=0}^{I-1} \mathcal{O}_T(-iK_T) \to T$$

is called the (global) *canonical covering*. Clearly, Y is either an abelian surface or a K3 surface with at worst Du Val singular points. We note also that π is unramified over the smooth part $T - \mathrm{Sing}\ T$.

We say that T is of *Type* A_m or D_n if Y has a singular point of Dynkin type A_m or D_n; T is of *actual Type* $(\oplus A_m) \oplus (\oplus D_n) \oplus (\oplus E_k)$ if Sing Y is of type $(\oplus A_m) \oplus (\oplus D_n) \oplus (\oplus E_k)$.

Around 1989, M. Reid and I. Naruki asked the second author about the uniqueness of rational log Enriques surface to Type D_{19}. The determinations of all isomorphism classes of rational log Enriques surfaces T of Type A_{19}, D_{19}, A_{18} and D_{18} have been done in [OZ1, 2] (see also [R1]). As a corrolary, the minimal

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