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Topological realization of the integer ring of local field

By

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1. Introduction

In the stable homotopy theory the complex cobordism theory MU and its p-local wedge summand BP are very important. The Morava K-theories $K(n)^*(\ )$ were invented by J. Morava in the early 1970s to understand the complex cobordism theory. In the present, however, from the work of Devinatz, Hopkins and Smith [2], [3], it becomes clear that Morava K-theories themselves play a very important and fundamental role in the stable homotopy theory.

Let p be a prime number. We consider in the p-local stable homotopy category. Morava K-theory $K(n)^*(\ )$ is a periodic cohomology of period $2(p^n-1)$. The coefficient ring is given by

$$K(n)_* = \boldsymbol{F}_p[v_n, v_n^{-1}],\ |v_n| = 2(p^n-1)$$

where v_n is the Hazewinkel generator. Let $\widehat{K(n)}$ be the p-adic Morava K-theory spectrum whose coefficient ring satisfies

$$\widehat{K(n)}_* = \mathcal{O}_K[u, u^{-1}],\ |u| = 2$$

where K is the degree n unramified extension of the p-adic number field $\boldsymbol{Q}_p$ and $\mathcal{O}_K$ is its integer ring. To simplify gradings, we use a formal (p^n-1)-th root u of v_n. It is known that the associated formal group law is the Lubin-Tate one [4]. Therefore $\widehat{K(n)}$ has intimate relation with the local class field theory. For example, the homotopy group of the Tate spectrum $t_{\boldsymbol{Z}/p}\widehat{K(n)}$ is the degree p^n-1 totally ramified abelian extension of K.

In this paper, as one aspect of this relation, we shall topologically realize the totally ramified abelian extensions of $\mathcal{O}_K$ which appear in the local class field theory by the method of Lubin-Tate formal group law. The main result (Theorem 3.3) is saying that we can construct a sequence of ring spectra and ring spectrum maps

$$\widehat{K(n)}(0) \xrightarrow{F_0} \widehat{K(n)}(1) \xrightarrow{F_1} \widehat{K(n)}(2) \xrightarrow{F_2} \cdots$$

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