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On the mod 3 homotopy type of the classifying space of a central product of $SU(3)$'s

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1. Introduction

Let $SU(3)$ be the compact Lie group of special unitary complex matrices of order 3. It is well known that the center of $SU(3)$, namely Γ, is isomorphic to $\mathbf{Z}/3$ and it is generated by the matrix (ω, ω, ω) where $\omega \in \mathbf{C}$ such that $\omega^3 = 1$ and $\omega \neq 1$. The compact Lie group $SU(3,3)$ is defined as the central product $SU(3) \times_{\mathbf{Z}/3} SU(3)$, i.e., as the quotient

$$SU(3,3) = SU(3) \times SU(3)/\Delta$$

where Δ is the subgroup of $SU(3) \times SU(3)$ generated by the elements (A, A) such that $A \in \Gamma$.

The group $SU(3,3)$ plays an important role when studying the homotopy type of the classifying space of the exceptional compact Lie group of rank 4, F_4, at primes greater than 3 (see [17] and [6]), and specially at the prime 3 (see [21]). This justify a deep study of the structure of $SU(3,3)$, as well as those of its classifying space $BSU(3,3)$, at the prime 3.

Our first result describes the mod 3 cohomology of $SU(3,3)$ as Hopf algebra.

Theorem 1.1. $H^*SU(3,3) = \mathbf{F}_3[y_2]/y_2^3 \otimes \Lambda_{\mathbf{F}_3}(x_1, x_3, x_3', x_5)$, *where subindex indicates degree. Moreover, the Hopf algebra structure is given by the reduced diagonal map*

a	x_1	x_3	x_3'	x_5	y_2
$\bar{\phi}(a)$	0	$y_2 \otimes x_1$	$y_2 \otimes x_1$	$y_2 \otimes (x_3' - x_3)$	0

Proof. See Section 3.

Then we calculate the mod 3 cohomology of the classifying space of $SU(3,3), BSU(3,3)$.

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