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1-cocycles on the group of diffeomorphisms II

By

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§1. Introduction

In this paper we consider 1-cocycles θ over finite or infinite configuration spaces on C^∞-manifolds M and natural representations connected with θ, which is exactly a continuation of the previous author's work [22]. Here the 1-cocycle is as a definition a $U(H)$-valued function on $X \times \mathrm{Diff}_0^*(M)$, which fulfills the so called cocycle equality, and $U(H)$ is the unitary group on a finite dimensional Hilbert space over C, $\mathrm{Diff}_0^*(M)$ is the connected component of the identity id in the group of diffeomorphisms with compact supports on M, and X is a collection B_M^n of all n-point subsets in M, or X is a space Γ_M of all infinite configurations on M. Historically in the first paper of Ismagilov [7] it is described that every irreducible unitary representation of the group $\mathrm{Diff}_0^*(T^1)$ with some additional properties is characterized as the natural representation $U_{\mu,\theta}$ with a suitable measure μ and a 1-cocycle θ on a configuration space or on an analogous one. After this natural representations frequently appeared in order to analyse or to construct unitary representations of $\mathrm{Diff}_0^*(M)$. (cf. [5], [6], [9], [25]) But the study of 1-cocycles have been rather neglected. Recently the author found that when the configuration space is M itself, the form of 1-cocycles is closely connected with a geometrical structure of M. (cf.[22]) That is, under an assumption that M is simply connected, every continuous[1] 1-cocycle θ has a canonical form consisting of only 1-coboundary and Jacobian term which are also the standard examples of 1-cocycles. Besides θ takes locally the canonical form without any assumption on M. Thus it is thought that a glueing of pieces actually determines the form of 1-cocycles on M. Combining these results with [7], we are led to a motivation of the present paper. That is: *Is the situation for a general configuration space similar with the previous one*?, and the answer is affirmative.

Let us explain our results in more detail. The next section begins with five definitions for regularity of 1-cocycles. Among them a notion of precontinuity is most fundamental. The principal part of this section is devoted to the study of precontinuous 1-cocycles $\tilde{\theta}(\bar{P},g)$ on $B_M^n \times \mathrm{Diff}_0^*(M)$. Since B_M^n is a quotient space of $\hat{M}^n := \{\hat{P}=(P_1, \cdots, P_n) \in M^n \mid {}^\forall i \neq j,\ P_i \neq P_j\}$ by an equivalence relation, we can always lift $\tilde{\theta}$ to $\hat{M}^n \times \mathrm{Diff}_0^*(M)$ as a symmetric one. So it is reasonable to start our study at precontinuous, however not necessarily symmetric, 1-cocycles on $\hat{M}^n \times \mathrm{Diff}_0^*(M)$.

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[1] The result is still true for precontinuous 1-cocycles as will be seen in the present paper.

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