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On the area of the complement of the invariant component of certain b-groups and on sequences of terminal regular b-groups

By

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Introduction

Let G be a finitely generated Fuchsian group of the first kind, and $\partial T(G)$ the Bers boundary of the Teichmüller space of G. Let χ_φ be the canonical isomorphism from G to the b-group corresponding to $\varphi \in \partial T(G)$ with suitable normalizations (cf. Section 1.2), and Δ_φ the invariant component of $\chi_\varphi(G)$. We know that *any* $\varphi \in \partial T(G)$ has a sequence $\{\varphi_m\}_{m=1}^{\infty}$ corresponding to terminal regular b-groups in $\partial T(G)$ such that φ_m converges to φ_0 and that the area of $C \setminus \Delta_{\varphi_m}$ tends to zero (cf. Remark (2) in Section 3.3). The main result of this paper is the following.

Theorem 1. *Let* $\{\varphi_n\}_{n=1}^{\infty} \subset \partial T(G)$ *be a sequence corresponding to terminal regular b-groups such that*

(a) *For any hyperbolic element* $g \in G$, *there exist* $\epsilon(g)$, $N(g) > 0$ *such that for* $n > N(g)$, *if* $\chi_{\varphi_n}(g)$ *is loxodronmic, then* $|\mathrm{tr}^2(\chi_{\varphi_n}(g)) - 4| \geq \epsilon(g)$, *and*

(b) *The Euclidean area of* $C \setminus \Delta_{\varphi_n}$ *tends to* 0 *as* $n \to \infty$.

Then every accumulation point of the sequence corresponds to a totally degenerate group.

This paper is organized as follows: In section 1, we fix our notations and recall some basic definitions and facts. Section 2 deals with a lower estimate of the complement of the invariant component of a b-group which contains triangle groups as component subgroups. This class of b-groups, by definition, involves the set of terminal regular b-groups. In Section 3, we give the proof of Theorem 1 and several remarks about our result.

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