**Numerical classification of singular fibers in genus 3 pencils**

By

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1. Introduction

Let \( \pi : X \to D \) be a proper surjective holomorphic map of a complex manifold \( X \) of dimension 2 to a small open disc \( D = \{ t \in \mathbb{C} \mid |t| < \varepsilon \} \). We assume that \( \pi \) is smooth over a punctured disc \( D' = D - \{0\} \). Moreover we assume that for every \( t \in D' \) the fiber \( X_t = \pi^{-1}(t) \) is a non-singular curve of genus \( g \) and that \( X \) contains no exceptional curves of the first kind. By \( L_0 \), we denote the effective divisor in \( X \) defined by the equation \( \pi = t (t \in D) \). We call the divisor \( L_0 \) the **singular fiber of \( \pi \).** For every \( t \in D' \) we call the divisor \( L_t \) a **generic fiber.** We write the singular fiber \( L_0 \) as

\[
(1.1) \quad L_0 = \sum_{i=1}^{r} n_i \Gamma_i,
\]

where \( \Gamma_i \) is an irreducible reduced component of \( L_0 \) and \( n_i \) is its multiplicity. By \( p(\Gamma_i) \), we denote the arithmetic genus of the component \( \Gamma_i \). The combination of integers \( \{r, n_i, p(\Gamma_i), \Gamma_i \cdot \Gamma_j \mid 1 \leq i < j \leq r\} \) is called a **numerical type of the singular fiber \( L_0).**

In the study of elliptic surfaces [2], Kodaira showed that there exist only ten types of singular fibers of pencils of curves of genus one. Iitaka [1] and Ogg [5] gave a numerical classification of singular fibers of curves of genus 2. Namikawa and Ueno [3], [4] classified their numerical types completely, constructed all their singular fibers and calculated the monodromies around them.

In our previous paper [6], we studied the numerical properties of singular fibers in pencils of curves of genus \( g \geq 2 \) and gave a method to classify all the numerical types of the singular fibers. In this article, by using this method, we give the complete numerical classification of singular fibers in pencils of curves of genus three.

If the number of irreducible components is more than one, we have \( \Gamma^2 < 0 \) and \( \Gamma \cdot K_X \geq 0 \), where \( K_X \) is a canonical divisor. If \( \Gamma \cdot K_X > 0 \), we call this component \( \Gamma \) a **trunk.** If \( \Gamma \cdot K_X = 0 \), then we have \( \Gamma^2 = -2 \). Thus we call this component \( \Gamma \) a **(-2)-curve.** Further we call a connected component consisting of \((-2)-curves in a singular fiber a **branch.** Our method of numerical classification

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