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On the Hopf algebra structure of the mod 3 cohomology of the exceptional Lie group of type E_6

By

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1. Introduction

Kono-Mimura [9] and Toda [13] determine $H^*(E_6;\mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$ the mod 3 Steenrod algebra. Kono [7] determines $H^*(AdE_6;\mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$ and simultaneously gives a new method to determine $H^*(E_6;\mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$. It is, however, very difficult to determine $\bar{\mu}^*(x_{17})$ where x_{17} is the generator of degree 17 in $H^*(E_6;\mathbf{Z}_3)$. (For a Hopf algebra, the reduced coproduct map is denoted by $\bar{\mu}^*$ in this paper.) In fact, a direct method is not found until now.

The main purpose of this paper is to give a direct method of the determination of $\bar{\mu}^*(x_{17})$. At the same time, $H^*(\tilde{E}_6;\mathbf{Z}_3)$ is determined as a Hopf algebra over $\mathscr{A}_3$ where $\tilde{E}_6$ is the 3-connective cover over E_6. For our purpose, in §2, we shall define five maps (which we call in this paper the adjoint maps) and state their properties. It should be emphasized that among these maps, what bring us improvement essentially are

$$\hat{\mathrm{ad}} : AdE_6 \wedge E_6 \to E_6$$

and

$$\check{\mathrm{ad}} : AdE_6 \wedge \tilde{E}_6 \to \tilde{E}_6.$$

In §3, we shall determine $H^*(AdE_6;\mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$ by a slightly different way from that of Kono [7]. In §4, we shall determine $H^*(E_6;\mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$. In §5, the last section, we shall prove the following.

Theorem 1.1. *As a Hopf algebra over $\mathscr{A}_3$, $H^*(\tilde{E}_6;\mathbf{Z}_3)$ is given as follows. As an algebra,*

$$H^*(\tilde{E}_6;\mathbf{Z}_3) = \mathbf{Z}_3[\tilde{y}_{18}] \otimes \Lambda(\tilde{x}_9, \tilde{x}_{11}, \tilde{x}_{15}, \tilde{x}_{17}, \tilde{y}_{19}, \tilde{y}_{23})$$

where $\deg \tilde{x}_k = \deg \tilde{y}_k = k$. *The coproducts are given by* $\bar{\mu}^*(\tilde{y}_{18}) = \tilde{x}_9 \otimes \tilde{x}_9$ *and* $\bar{\mu}^*(z) = 0$ $(z = \tilde{x}_k, \tilde{y}_{19}, \tilde{y}_{23})$. *The cohomology operations are given by*

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