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A remark on Baker operations on the elliptic cohomology of finite groups

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Introduction

Let G be a finite group and BG be its classifying space. In [18] it is shown that every element of $Ell^{\text{even}}(BG)$ yields a certain p-adic limit of Thompson series via elliptic character. We hope that this fact will shed light on still unknown geometric construction of elliptic cohomology and hence the study of $Ell^*(BG)$ in connection with moonshine phenomena would be important.

In theory of Thompson series we have certain Hecke operators constructed by G. Mason which are related to usual Hecke operators on modular forms (see [11]). The purpose of this note is to prove that the stable operations on elliptic cohomology constructed by A. Baker in [3] act on $Ell^{\text{even}}(BG)$ as Mason's Hecke operators act on Thompson series (Theorem 3.1). The proof of this fact is pretty easy. First we review the construction of elliptic character and Baker operations and describe the composition of these natural transformations (Proposition 1.3). By using this description and a certain explicit formula given in [18] we can easily prove Theorem 3.1 (§3).

1. Elliptic character and Baker operation

We begin by considering a general construction of natural transformations of cohomology theories obtained by Landweber exact functor theorem (Landweber exact cohomologies for short). Let $R_*(?)=R_*\otimes_{MU_*}MU_*(?)$ and $S_*(?)=S_*\otimes_{MU_*}MU_*(?)$ be Landweber exact homologies. Let

$$\Lambda : R_*\otimes_{MU_*}MU_*(MU)\to S_*$$

be a right MU_*-module map. Then we can define a natural transformation

$$R_*(X)\to S_*(X)$$

as the composite map

$$R_*\otimes_{MU_*}MU_*(X)\xrightarrow{R_*\otimes\psi_X} R_*\otimes_{MU_*}MU_*(MU)\otimes_{MU_*}MU_*(X)$$

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