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# Removable singularities for semilinear degenerate elliptic equations and its application

Dedicated to Professor Norio Shimakura on the occasion of his sixtieth birthday

By

Toshio Horiuchi

## 0. Introduction

Let $N \geq 1$ and $p > 1$. Let $\Omega$ be a bounded open set with smooth boundary and $F$ be a compact set satisfying $F \subset \Omega \subset \boldsymbol{R}^N$. We also set $\Omega' = \Omega \setminus \partial F$, where $\partial F = F \setminus \operatorname{Int} F$. We assume that the measure of $\partial F$ is zero. Define

$$P = -\operatorname{div}(A(x)\nabla \cdot), \tag{0-1}$$

where $A(x) \in C^1(\Omega')$ is positive in $\Omega \setminus F$ and vanishes in $\operatorname{Int} F$. First we shall consider removable singularities of solutions for degenerate semilinear elliptic equations. Assume that $u \in C^0(\Omega') \cap C^2(\Omega \setminus F)$ satisfies the differential inequality

$$Pu + B(x)Q(u) \leq C(x), \quad \text{in } \Omega', \tag{0-2}$$

for some nonnegative functions $B(x)$ and $C(x)$. Here $Q(t)$ is continuous and strictly monotone increasing on $\boldsymbol{R}$ satisfying the growth condition (1-6). For instance we can adopt $|t|^{p-1}t$ with $p > 1$ and $(e^{|t|} - 1)\operatorname{sgn}(t)$ for $Q(t)$. Then we shall show under some additional conditions on $A(x)$, $B(x)$, $C(x)$ and $Q(t)$ that

$$\limsup_{x \to \partial F} u(x) < +\infty. \tag{0-3}$$

From this result we can deduce that if $u \in C^0(\Omega') \cap C^2(\Omega \setminus F)$ satisfies

$$Pu + B(x)Q(u) = f(x), \quad \text{in } \Omega' \tag{0-4}$$

for $f/B \in L^\infty(\Omega)$, then there is a bounded function in $\Omega$ which coincides with $u$ in $\Omega' = \Omega \setminus \partial F$.

This result was established by H. Brezis and L. Veron, under the assumptions that $F$ consists of finite points, $Q(t) = |t|^{p-1}t$ and $A(x)$, $B(x)$, $C(x)$ are positive constants. More precisely they proved in [BV] that if $u$ satisfies (0-2) with some

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