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A certain class of distribution-valued additive functionals I —for the case of Brownian motion

By

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1. Introduction

Let B_s be a one-dimensional Brownian motion and T be a distribution which belongs to the class $\mathscr{D}'^{1}_{L^2_{loc}}$. M. Fukushima has proposed a definition of the integral $\int_0^t T(B_s)ds$ via Ito's formula and showed that the integral is a continuous additive functional of zero energy ([3]).

T. Yamada [11] and M. Yor [13] studied concretely principal values of Brownian local time which are typical examples in the class of additive functionals of zero energy.

It is well known that there is a one-to-one correspondence between the class of positive continuous additive functionals of d-dimensional Brownian motion and the class of Revuz measures ([7], [8]).

R. Bass [1] showed that additive functionals $A(a, t, \omega)$ for d-dimensional Brownian motion are jointly continuous in a and t, $a.s.$ and represented $A(a, t, \omega)$ as d-dimensional analogue to the Ito and Mckean [5] that states that any additive functional A_t of one-dimensional Brownian motion can be represented as

$$A_t = \int L_t^y \mu(dy),$$

where L_t^y is the local time at y for the one-dimensional Brownian motion and μ is the measure corresponding to A_t.

T. Yamada showed that any continuous additive functional of zero energy has a representation via convolution-type transform of the local time in the case of one-dimensional Brownian motion and generalized a representation formula given by R. Bass in the case of multi-dimensional Brownian motion ([12]).

In this paper, we show that $A_T(a : t, \omega) = \int_0^t T(X_s - a)ds$ is a continuous additive functional for some $T \in H_p^\beta$, where X_s is d-dimensional Brownian motion and this additive functional has jointly continuous modification in a and t, $a.s.$ and has zero energy.

Our method is very simple. It is principally based on the Fourier transform theory in distribution sense. The concrete estimate of the characteristic function

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