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On steady Stokes and Navier-Stokes problems with zero velocity at infinity in a three-dimensional exterior domain

By

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1. Introduction

In the paper we study the exterior Stokes and Navier-Stokes problems with zero conditions at infinity in weighted function spaces. Let us formulate these problems. Let Ω be an exterior domain in $\mathbf{R}^3$ (i.e. $\Omega=\mathbf{R}^3\backslash\bar{G}$, where G is a bounded domain). Without any loss of generality we can assume that the Cartesian coordinate system in $\mathbf{R}^3$ is chosen so, that the origin lies outside $\bar{\Omega}$ i.e. the point $x=0$ belongs to G. We also assume the boundary $\partial\Omega$ to be a smooth compact manifold. In Ω we consider the Stokes

$$-\nu\Delta\mathbf{v}+\nabla p=\mathbf{f},\quad x\in\Omega,$$

$$\nabla\cdot\mathbf{v}=g,\quad x\in\Omega, \tag{1.1}$$

and Navier-Stokes

$$-\nu\Delta\mathbf{v}+(\mathbf{v}\cdot\nabla)\mathbf{v}+\nabla p=\mathbf{f},\quad x\in\Omega,$$

$$\nabla\cdot\mathbf{v}=0,\quad x\in\Omega, \tag{1.2}$$

systems of equations with the boundary conditions

$$\mathbf{v}=\mathbf{h},\quad x\in\partial\Omega. \tag{1.3}$$

Moreover, we assume the velocity field $\mathbf{v}$ to vanish at infinity

$$\lim_{|x|\to\infty}\mathbf{v}(x)=0. \tag{1.4}$$

In (1.1)-(1.4) $x=(x_1,x_2,x_3)\in\mathbf{R}^3$, $\nabla=(\partial/\partial x_1,\partial/\partial x_2,\partial/\partial x_3)$, $\mathbf{v}=(v_1,v_2,v_3)$ and p are the velocity field and the pressure function in the flow, ν is the coefficient of viscosity, $\mathbf{f}$ and $\mathbf{h}$ are given vector functions in $\mathbf{R}^3$ and g is a given scalar function. By "$\cdot$" we denote the scalar product in $\mathbf{R}^3$.

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