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Uniqueness in inverse hyperbolic problems
—Carleman estimate for boundary value problems—

By

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1. Introduction

We give sharp Carleman estimates including boundary conditions to show the unique continuation across a lateral boundary for hyperbolic equations, and we show the uniqueness in hyperbolic inverse problems by the use of the above unique continuation.

T. Carleman [4] proposed a weighted inequality to show the uniqueness in Cauchy problems to which Holmgren's theorem are not applicable, and we call this type of weighted estimates the Carleman estimates. The Carleman estimate has been playing an important role to show the uniqueness not only in Cauchy problems but in inverse problems. Especially for inverse hyperbolic problems, the uniqueness is one of the most interesting problems in this field, and many researchers study applications of the Carleman estimate; e.g. Bukhgeĭm [2], Bukhgeĭm and Klibanov [3], Isakov [6], Lavrent'ev, Romanov and Shishat-skiĭ [9], Yamamoto [19] etc.

The author gives a new type of the Carleman estimates in the present paper, and he shows sharp results in the uniqueness. The main interest of this research lies in an inverse problem to identify unknown coefficients of the wave equation from measurement on a lateral boundary. The problem is attractive for many researchers, since it is a mathematical model in geophysics to find properties of geophysical media by observation of wave fields on a part of the surface of the Earth. We wish to know conditions for the uniqueness of solutions, but the uniqueness has not been shown for the case observation is done on a part of a boundary. We show sharp estimates to give conditions for the uniqueness to this case. Proofs of uniqueness theorems of inverse problems are based on the following two points;
(1) the Bukhgeĭm-Klibanov method presented in [3],
(2) Carleman estimates near the boundary for boundary value problems.
We remark the method (1) is an application of the Carleman estimate to inverse problems and effective for various inverse problems to determine coefficients in the equations for which the Carleman estimate holds. Since the Carleman estimate depends essentially on a relation between the type of differential equations and the shape of a domain, and many serious difficulties arise in particular for hyperbolic

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