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Behavior of rational curves of the minimal degree in projective space bundle in any characteristic

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In this paper we investigate the behavior of rational curves of the minimal degree in a projective space bundle. Particularly we try to generalize the theory of adjoint bundle of ample vector bundles to any characteristic. In characteristic zero the classifications of their adjoint bundles are made by many authors [F2], [F3], [Io], [PW], [YZ]. Then a main tool is the contraction theorem due to Mori-Kawamata and Kobayashi-Ochiai theorem. In particular they heavily depend on generalized Kodaira-Vanishing theorem in characteristic zero.
On the other hand considering Theorem by [YZ] in any characteristic, we have

Theorem 2.6. *Let X be an n-dimensional smooth projective variety defined over an algebraically closed field of any characteristic and let E be an ample vector bundle on X. Assume that $K_X + c_1(E)$ is not nef. Then we have*

1) *If* $\operatorname{rank} E = n \geq 2$, *then each line bundle of X is numerically equivalent to aL with an integer a where $L := -K_X - c_1(E)$ is an ample line bundle. In particular $-K_X$ is numerically equivalent to $(n+1)L$.*

2) *If* $\operatorname{rank} E = n - 1 \geq 3$, *$X$ has two cases*:

I) $NS(X) \times_{\mathbf{Z}} \mathbf{Q} \cong \mathbf{Q}$ *and X is Fano.*

II) *There is a generically bijective and finite morphism $\phi : D \to X$ from a projective variety D to X where $\psi : D \to C$ is a fiber space over a projective curve C. Moreover for each point c in C $\phi(\psi^{-1}(c))$ is a divisor swept out by rational curves parameterised by $(2n-4)$-dimensional divisor of Y.* (*See Proposition* 2.3.2 *for* Y)

Note that if E is spanned then the same conclusion (Theorems 4.7.2 and 6.16) as in Theorems 1 and 2 [YZ] is obtained. Namely X is one of $\mathbf{P}^n$, hyperquadric and scroll over a smooth curve. It is a corollary of Theorem 4.1 A, B stated just below.

Now when we study the adjoint bundle of ample vector bundle (X, E), we see that the essential point is 1) to show the existence of extremal rational

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