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The theory of second-order differential equations based on Finsler geometry

By

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The first author has felt a great interest in the geometrical approach to the theory of nonlinear dynamical systems on the basis of Finsler metrics ([4], [5]) and gave a lecture with the title "Finsler's metrics of phase space of nonlinear dynamical systems and their applications" at the International Conference on Differential Geometry and its Applications, Bucharest, 1992 (Tensor, N. S., 53, x vi, ↓ 3). The second author participated in the Conference as a member of the Committee of Organization. Since then, the intimate contacts between the present authors have continued through the applications of Finsler geometry in the nonlinear dynamical systems.

Finsler geometry has founded by P. Finsler (1918) as the differential-geometrical development of variation calculus. Consequently the theory of geodesics should be one of the essential fields of the differential geometry of Finsler spaces.

Closely related to the behavior of geodesics, the conception of Finsler space with rectilinear extremals has been proposed and studied by P. Funk and L. Berwald. In particular, Berwald (1941) and the second author [7] have established the necessary and sufficient conditions for a Finsler space to be with rectilinear extremals, and the second author [8] has proved that the transformation of rectilinear coordinate systems is projective.

Directly motivated by the behavior of geodesic equations of two-dimensional Riemannian space and Berwald space, the second author with S. Bácsó [3] have found the notion of Finsler space of Douglas type and investigated the differential-geometrical aspects of the special Finsler spaces. In the two-dimensional case those spaces are characterized by the remarkable fact that the right-hand side of the geodesic equation $y'' = f(x, y, y')$ is a polynomial in y' of degree at most three.

In the recent paper [12] the second author showed that a projectively equivalent class of two-dimensional Finsler spaces is associated to a given second-order differential equation $y'' = f(x, y, y')$ of the normal form such that it is a differential equation of geodesics of every space of this class. Consequently the behavior of $y'' = f(x, y, y')$ is directly connected with that of the spaces.

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