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On a Certain Extended Galois Action on the Space of Arithmetic Modular Forms with respect to a Unitary Group

By

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Introduction

In his work [8, Theorem 1.5], G. Shimura proved the existence of a certain Galois action on the graded ring of Hilbert modular forms. A holomorphic Hilbert modular form f with respect to $\mathrm{SL}(2, F)$ (where F is a totally real algebraic number field of finite degree) can be expressed as a Fourier series of complex variables $u_1, \ldots, u_l$

$$f(u_1, \ldots, u_l) = \sum_x c_x \exp\left(2\pi\sqrt{-1}\sum_{\mu=1}^{l} x_\mu u_\mu\right), \tag{0.1}$$

where the coefficients $c_x \in \mathbb{C}$ and x runs over a lattice. It is shown first that, for any $\sigma \in \mathrm{Aut}(\mathbb{C})$, there exists a holomorphic modular form f^σ whose Fourier expansion is

$$f^\sigma(u_1, \ldots, u_l) = \sum_x c_x^\sigma \exp\left(2\pi\sqrt{-1}\sum_{\mu=1}^{l} x_\mu u_\mu\right). \tag{0.2}$$

A Hilbert modular form with respect to $\mathrm{SL}(2, F)$ has the weight in $\sum_{v\in\mathbf{a}} \mathbb{Z}\cdot v$, where $\mathbf{a}$ is the set of all embeddings of F into $\mathbb{R}$. If f is of weight $k = \sum_{v\in\mathbf{a}} k_v \cdot v$, then f^σ is of weight $k^\sigma = \sum_{v\in\mathbf{a}} k_v \cdot v\sigma$. It is also shown that, there exists a certain closed subgroup $\mathfrak{G}$ of $\mathrm{GL}(2, F_A) \times \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ which acts on the graded ring of meromorphic Hilbert modular forms which can be expressed as a quotient of holomorphic Hilbert modular forms with $\overline{\mathbb{Q}}$-rational Fourier coefficients. An important aspect here is that the action of $\mathfrak{G}$ on Hilbert modular forms of weight 0 coincides with that of $\mathfrak{G}$ in the theory of canonical models constructed in [2].

In this paper we shall study such a Galois action on modular forms in the case of unitary groups. On unitary groups, modular forms have no Fourier

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