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Some estimates of the logarithmic Sobolev constants on manifolds with boundary and an application to the Ising models

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1. Introduction

In this article we will give some estimates of the logarithmic Sobolev constant in terms of geometric quantities under the Neumann boudary condition and will give an application to the logarithmic Sobolev inequality for the Ising models.

Let M be a d-dimensional, smooth, compact and connected Riemannian manifold with smooth boundary ∂M and let m be the Riemannian measure. For a potential function $U \in C^\infty(M)$, set $dm^U = e^{-U}dm$ and L^U by

$$L^U f = \triangle f - (\nabla U | \nabla f)$$

for $f \in C^\infty(M)$ satisfying the Neumann boundary condition. In the first part of this article we will consider the spectral gap and the logarithnic Sobolev inequality for L^U (or equivalently, for $\int_M (\nabla f, \nabla g) dm^U$) and give some estimates of the spectral gap constant and logarithmic Sobolev constant. Note that that the logarithmic Sobolev inequality (hence the spectral gap, too) holds is proved by using the strict positivity of the heat kernel (see Chapter VI of Deuschel and Stroock [3]). In the latter part we will consider the Gibbs measures on $M^{\mathbf{Z}^\nu}$ determined by a finite range and shift-invariant potential and will apply those results for finite dimensional manifolds to the Ising models.

In fact, our article is a generalization of Deuschel and Stroock [2] and based on it. They argued those problems on smooth manifolds (without boundary) by searching for constants which make the Bakry-Emery criterion [1] hold. In our case a similar argument holds with a slight modification by adding a certain term which includes the second fundamental form. When the second fundamental form is non-negative our results in this article are the same as those in [2]. So we will prepare some estimates of the second fundamental form in Lemma 2.4 to have those estimates of the logarithmic Sobolev constant in

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