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Vertical Gromov-Witten invariants of flag bundles

By

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Introduction

Let G be a complex semi-simple connected Lie group and P its parabolic subgroup. Then the maximal torus T of G acts on the homogeneous space G/P and this action can be naturally lifted to the moduli space (stack) of the stable maps $\overline{\mathcal{M}}_{g,n}(G/P, \beta)$, where β is an element in $H_2(G/P, \mathbf{Z})$. In this paper, we investigate the T-equivariant Gromov-Witten invariants of G/P and vertical invariants of flag bundles. We mainly consider the invariants of genus zero. We will discuss on higher genus invariants briefly in Section 6. We will work only over the ground field $\mathbf{C}$ throughout this paper.

Kontsevich used the fixed point localization method for the first time to compute the Gromov-Witten invariants of the projective space and its hypersurfaces in his paper [16]. In Section 5.2 of [16], he mentioned that his computational scheme works well also for homogeneous spaces, and we will follow his method to give a formula of (gravitational) Gromov-Witten invariants of the homogeneous space G/P. The fixed point localization method enables us to obtain the information of the equivariant cohomology $H_T^*(\overline{\mathcal{M}}_{0,n}(G/P, \beta))$ from the data on the fixed points of the action of T. In our case, the set of fixed points $\overline{\mathcal{M}}_{0,n}(G/P, \beta)^T$ can be described in terms of the Bruhat ordering of the Weyl group W of G. Integration of an element in $H_T^*(\overline{\mathcal{M}}_{0,n}(G/P, \beta))$ can be expressed as a sum of the contributions from the components of the fixed locus. Such formula is known as Bott's fixed point formula. Then, the Gromov-Witten invariants of the flag variety G/B can be computed effectively by using Bott's fixed point formula for smooth stacks.

The fixed point localization method was also used in the proof of the Mirror Theorem for complete intersections in projective space by Givental [11]. Kim also obtained the Mirror Theorem when the ambient space is a homogeneous space [15]. The explanation of Kontsevich's method and Givental's proof of the Mirror Theorem can be found in [5].

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