On Absolute continuity of the Gibbs measure
under translations

By

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1. Introduction

The question of absolute continuity of measures under transformations of
the spaces has been treated in many cases.

One typical example is the case of infinite product measures on $\mathbb{R}^\infty$: Kakutani [10] proved that two infinite product measures are either equivalent or mutually singular. Later, Shepp [23] showed that $l^2$ is the space of admissible
shifts for infinite product measures on $\mathbb{R}^\infty$.

Such properties, dichotomy between equivalence and mutual singularity,
and characterization of admissible shifts, were also shown in some different
cases: in the case of Gaussian measures [4], [7], [9], and in the case of the
Brownian motion measure and the pinned Brownian motion measure over the
compact Lie group [1], [14], [22], [24].

In this paper, we treat Gibbs measures of unbounded lattice spin systems.
We will show that each element of $l^2(\mathbb{Z}^d)$ is an admissible direction for the Gibbs
measure if the second derivative of the potential satisfies a certain integrability
condition. We will also show that, in one dimensional case, dichotomy between
equivalence and mutual singularity holds for the Gibbs measure if the self-
potential is uniformly convex. In this case, $l^2(\mathbb{Z})$ is the space of admissible
shifts.

The organization of this paper is as follows. In Section 2, we fix some
notation and collect known results about Gibbs measures which we need in the
later sections. In Section 3, we show that each element of $l^2(\mathbb{Z}^d)$ is an admissible
direction for the Gibbs measure. Lemma 3.2, which is based on (3.5), plays
an essential role. We give an expression for the Radon-Nikodym derivative
in terms of formal Hamiltonian which is defined reasonably. In Section 4, we
treat one dimensional case. We show that, in the case of uniformly convex
self-potentials, the Hellinger integral vanishes if the transformation is a shift
by $h \notin l^2(\mathbb{Z})$.