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On Absolute continuity of the Gibbs measure under translations

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1. Introduction

The question of absolute continuity of measures under transformations of the spaces has been treated in many cases.

One typical example is the case of infinite product measures on $\mathbb{R}^\infty$: Kakutani [10] proved that two infinite product measures are either equivalent or mutually singular. Later, Shepp [23] showed that l^2 is the space of admissible shifts for infinite product measures on $\mathbb{R}^\infty$.

Such properties, dichotomy between equivalence and mutual singularity, and characterization of admissible shifts, were also shown in some different cases: in the case of Gaussian measures [4], [7], [9], and in the case of the Brownian motion measure and the pinned Brownian motion measure over the compact Lie group [1], [14], [22], [24].

In this paper, we treat Gibbs measures of unbounded lattice spin systems. We will show that each element of $l^2(\mathbb{Z}^d)$ is an admissible direction for the Gibbs measure if the second derivative of the potential satisfies a certain integrability condition. We will also show that, in one dimensional case, dichotomy between equivalence and mutual singularity holds for the Gibbs measure if the self-potential is uniformly convex. In this case, $l^2(\mathbb{Z})$ is the space of admissible shifts.

The organization of this paper is as follows. In Section 2, we fix some notation and collect known results about Gibbs measures which we need in the later sections. In Section 3, we show that each element of $l^2(\mathbb{Z}^d)$ is an admissible direction for the Gibbs measure. Lemma 3.2, which is based on (3.5), plays an essential role. We give an expression for the Radon-Nikodym derivative in terms of formal Hamiltonian which is defined reasonably. In Section 4, we treat one dimensional case. We show that, in the case of uniformly convex self-potentials, the Hellinger integral vanishes if the transformation is a shift by $h \notin l^2(\mathbb{Z})$.

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