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3-graded decompositions of exceptional Lie algebras $\mathfrak{g}$ and group realizations of $\mathfrak{g}_{ev}, \mathfrak{g}_0$ and $\mathfrak{g}_{ed}$ Part I, $G = G_2, F_4, E_6$

By

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The ν-graded decomposition of simple Lie algebras $\mathfrak{g}$, $\mathfrak{g} = \sum_{k=-\nu}^{\nu} \mathfrak{g}_k, [\mathfrak{g}_i, \mathfrak{g}_j] \subset \mathfrak{g}_{i+j}$, has been studied by many mathematicians. Firstly the case of $\nu = 1$ was studied by S. Kobayashi–T. Nagano [4]. The case of $\nu = 2$, S. Kaneyuki [3] classified and determined the types of subalgebras $\mathfrak{g}_{ev}, \mathfrak{g}_0$ of $\mathfrak{g}$ and in the exceptional case, S. Gomyo [1] gave explicit realization of each $\mathfrak{g}_k$, I. Yokota [8], [9], [10] gave group realization of $\mathfrak{g}_{ev}, \mathfrak{g}_0$. Now, recently M. Hara [2] classified the 3-graded decomposition of simple Lie algebras $\mathfrak{g}$,

$$\mathfrak{g} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_{-2} \oplus \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3$$

and determined the types of subalgebras $\mathfrak{g}_{ev} = \mathfrak{g}_{-2} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_2, \mathfrak{g}_0$ and $\mathfrak{g}_{ed} = \mathfrak{g}_{-3} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_3$ of $\mathfrak{g}$. The following table is the results of $\mathfrak{g}_{ev}, \mathfrak{g}_0, \mathfrak{g}_{ed}$ for the exceptional Lie algebras $\mathfrak{g}$ of type G_2, F_4 and E_6.

$\mathfrak{g}$	$\dim \mathfrak{g}_1, \dim \mathfrak{g}_2, \dim \mathfrak{g}_3$ $\mathfrak{g}_0$	$\mathfrak{g}_{ev}$ $\mathfrak{g}_{ed}$
${\mathfrak{g}_2}^C$	2, 1, 2 $C \oplus \mathfrak{sl}(2, C)$	$\mathfrak{sl}(2, C) \oplus \mathfrak{sl}(2, C)$ $\mathfrak{sl}(3, C)$
$\mathfrak{g}_{2(2)}$	2, 1, 2 $\boldsymbol{R} \oplus \mathfrak{sl}(2, \boldsymbol{R})$	$\mathfrak{sl}(2, \boldsymbol{R}) \oplus \mathfrak{sl}(2, \boldsymbol{R})$ $\mathfrak{sl}(3, \boldsymbol{R})$
${\mathfrak{f}_4}^C$	12, 6, 2 $C \oplus \mathfrak{sl}(2, C) \oplus \mathfrak{sl}(3, C)$	$\mathfrak{sl}(2, C) \oplus \mathfrak{sp}(3, C)$ $\mathfrak{sl}(3, C) \oplus \mathfrak{sl}(3, C)$
$\mathfrak{f}_{4(4)}$	12, 6, 2 $\boldsymbol{R} \oplus \mathfrak{sl}(2, \boldsymbol{R}) \oplus \mathfrak{sl}(3, \boldsymbol{R})$	$\mathfrak{sl}(2, \boldsymbol{R}) \oplus \mathfrak{sp}(3, \boldsymbol{R})$ $\mathfrak{sl}(3, \boldsymbol{R}) \oplus \mathfrak{sl}(3, \boldsymbol{R})$
${\mathfrak{e}_6}^C$	18, 9, 2 $C \oplus \mathfrak{sl}(2, C) \oplus \mathfrak{sl}(3, C) \oplus \mathfrak{sl}(3, C)$	$\mathfrak{sl}(2, C) \oplus \mathfrak{sl}(6, C)$ $\mathfrak{sl}(3, C) \oplus \mathfrak{sl}(3, C) \oplus \mathfrak{sl}(3, C)$
$\mathfrak{e}_{6(6)}$	18, 9, 2 $\boldsymbol{R} \oplus \mathfrak{sl}(2, \boldsymbol{R}) \oplus \mathfrak{sl}(3, \boldsymbol{R}) \oplus \mathfrak{sl}(3, \boldsymbol{R})$	$\mathfrak{sl}(2, \boldsymbol{R}) \oplus \mathfrak{sl}(6, \boldsymbol{R})$ $\mathfrak{sl}(3, \boldsymbol{R}) \oplus \mathfrak{sl}(3, \boldsymbol{R}) \oplus \mathfrak{sl}(3, \boldsymbol{R})$

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