J. Math. Kyoto Univ. (JMKYAZ)
41-4 (2002), 809–827

The invariance of analytic assembly maps under the groupoid equivalence

By

Jae-Kwan Shim*

Introduction

The original motivation for the work of Baum and Connes ([1], [2]) was to construct a geometric or topological version $K^*(M,G)$ of the K-theory group $K_*(C_r^*(M \rtimes G))$, where $C_r^*(M \rtimes G)$ is the reduced C^*-algebra associated to the Lie group action of G on a manifold M. $K^*(M,G)$ is much easier to calculate than $K_*(C_r^*(M \rtimes G))$ since there are geometric and topological tools available for the calculation of $K^*(M,G)$. The cocycles of $K^*(M,G)$ are triples (Z,σ,f), where Z is a proper smooth G-manifold, $f : Z \to M$ is a G-equivariant smooth submersion, and σ is a G-equivariant symbol along the fibers of f. The (reduced) analytic assembly map $\mu_r : K^*(M,G) \to K_*(C_r^*(M \rtimes G))$ is defined as follows: on each fiber the symbol σ gives an elliptic operator D_x, and $\mu_r(Z,\sigma,f)$ is the index of the family (D_x). It is conjectured by P. Baum and A. Connes that this map is an isomorphism.

It has many important implications in topology and analysis. For instance, the rational injectivity of μ_r implies the Novikov conjecture on the homotopy invariance of higher signatures ([11]), and the Gromov-Lawson-Rosenberg conjecture on manifolds admitting metrics of positive scalar curvature ([17]). The surjectivity of μ_r implies the generalized Kadison conjecture on the nonexistence of projections in $C_r^*(\Gamma)$ where Γ is a torsion-free discrete group.

In [6], A. Connes sketched the construction of the analytic assembly map for a general smooth groupoid $\mathcal{G}$,

$$K_{top}^*(\mathcal{G}) \xrightarrow{\mu_{\mathcal{G}}} K_*(C^*(\mathcal{G})).$$

Then he conjectured that the composition of $\mu_{\mathcal{G}}$ with the canonical map from $K_*(C^*(\mathcal{G}))$ to $K_*(C_r^*(\mathcal{G}))$, which will be called the reduced analytic assembly map, is an isomorphism. This conjecture will be called the *Baum-Connes conjecture for $\mathcal{G}$*.

Communicated by Prof. K. Ueno, May 24, 2000
Revised June 30, 2001
*This work was supported by the Brain Korea 21 Project.