

# Mod 3 homotopy uniqueness of $BF_4$

By

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## 1. Introduction

Let  $F_4$  be the exceptional compact Lie group of rank 4, and denote by  $BF_4$  its classifying space. Previous work about homotopy uniqueness of classifying spaces of compact Lie groups by Dwyer-Miller-Wilkerson [6], and Notbohm [16], shows that this classifying space is determined, up to completion, by its mod  $p$  cohomology at primes greater than 3, that is, if  $X$  is a  $p$ -complete space ( $p > 3$ ) such that  $H^*(X; \mathbb{F}_p)$  is isomorphic to  $H^*(BF_4; \mathbb{F}_p)$  as  $\mathcal{A}_p$ -algebras, then  $X$  is homotopy equivalent to  $BF_4$  up to  $p$ -completion. At the prime 3,  $BF_4$  has torsion and its mod 3 cohomology was calculated by Toda [20]. As an algebra:

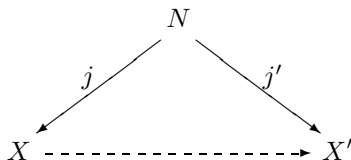
$$H^*(BF_4; \mathbb{F}_3) = \mathbb{F}_3[t_4, t_8, t_{20}, t_{26}, t_{36}, t_{48}] \otimes \Lambda_{\mathbb{F}_3}(t_9, t_{21}, t_{25})/R,$$

where  $R$  is an ideal generated by  $t_4t_9$ ,  $t_8t_9$ ,  $t_4t_{21}$ ,  $t_9t_{20} + t_8t_{21}$ ,  $t_9t_{20} + t_4t_{25}$ ,  $t_{26}t_4 + t_{21}t_9$ ,  $t_8t_{25}$ ,  $t_{26}t_8 - t_{25}t_9$ ,  $t_{20}t_{21}$ ,  $t_{20}t_{25}$ ,  $t_{26}t_{20} - t_{21}t_{25}$  and  $t_{20}^3 - t_4^3t_{48} - t_8^3t_{36} + t_{20}^2t_8^2t_4$ . In this note we prove that  $BF_4$  is determined up to completion by its cohomology at the torsion prime 3, as well.

**Theorem 1.1.** *Let  $X$  be a 3-complete space such that  $H^*(X; \mathbb{F}_3)$  is isomorphic to  $H^*(BF_4; \mathbb{F}_3)$  as  $\mathcal{A}_3$ -algebras. Then  $X$  is homotopy equivalent to  $BF_4$  up to 3-completion.*

*Proof.* See Section 2. □

A different question is whether or not a compact Lie group or  $p$ -compact group is  $N$ -determined (see [12] and [18]): Let  $X$  be a  $p$ -compact group and  $j: N \rightarrow X$  its maximal torus normalizer. Then  $X$  is said to be  $N$ -determined if any diagram




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