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Mod 3 homotopy uniqueness of BF_4

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1. Introduction

Let F_4 be the exceptional compact Lie group of rank 4, and denote by BF_4 its classifying space. Previous work about homotopy uniqueness of classifying spaces of compact Lie groups by Dwyer-Miller-Wilkerson [6], and Notbohm [16], shows that this classifying space is determined, up to completion, by its mod p cohomology at primes greater than 3, that is, if X is a p-complete space $(p > 3)$ such that $H^*(X;\mathbb{F}_p)$ is isomorphic to $H^*(BF_4;\mathbb{F}_p)$ as $\mathcal{A}_p$-algebras, then X is homotopy equivalent to BF_4 up to p-completion. At the prime 3, BF_4 has torsion and its mod 3 cohomology was calculated by Toda [20]. As an algebra:

$$H^*(BF_4;\mathbb{F}_3) = \mathbb{F}_3[t_4, t_8, t_{20}, t_{26}, t_{36}, t_{48}] \otimes \Lambda_{\mathbb{F}_3}(t_9, t_{21}, t_{25})/R,$$

where R is an ideal generated by t_4t_9, t_8t_9, t_4t_{21}, $t_9t_{20} + t_8t_{21}$, $t_9t_{20} + t_4t_{25}$, $t_{26}t_4 + t_{21}t_9$, t_8t_{25}, $t_{26}t_8 - t_{25}t_9$, $t_{20}t_{21}$, $t_{20}t_{25}$, $t_{26}t_{20} - t_{21}t_{25}$ and $t_{20}^3 - t_4^3t_{48} - t_8^3t_{36} + t_{20}^2t_8^2t_4$. In this note we prove that BF_4 is determined up to completion by its cohomology at the torsion prime 3, as well.

Theorem 1.1. *Let X be a 3-complete space such that $H^*(X;\mathbb{F}_3)$ is isomorphic to $H^*(BF_4;\mathbb{F}_3)$ as $\mathcal{A}_3$-algebras. Then X is homotopy equivalent to BF_4 up to 3-completion.*

Proof. See Section 2. □

A different question is whether or not a compact Lie group or p-compact group is N-determined (see [12] and [18]): Let X be a p-compact group and $j\colon N \longrightarrow X$ its maximal torus normalizer. Then X is said to be N-determined if any diagram

$$\begin{tikzcd} & N \arrow[dl, "j"'] \arrow[dr, "j'"] & \\ X \arrow[rr, dashed] & & X' \end{tikzcd}$$

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