J. Math. Kyoto Univ. (JMKYAZ)
41-4 (2002), 669–692

A bifurcation phenomenon for the periodic solutions of a semilinear dissipative wave equation

By

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1. Introduction and main result

In this paper, we consider the time periodic solutions of a following semilinear dissipative wave equation

$$u_{tt} - c^2 u_{xx} + \mu u_t + u^3 = f(t, x), \quad t, x \in R, \tag{1.1}$$

with periodic boundary condition

$$u(t, x) = u(t, x + L), \quad t, x \in R. \tag{1.2}$$

Here c and μ are positive constants and $f(t, x)$ is a given external force, which is T-periodic in t. It is known that for any periodic external force, there exists at least one T-periodic solution of (1.1) with (1.2). Moreover, if $f(t, x)$ is suitably small, then any time periodic solution of (1.1) with (1.2) is unique and asymptotically stable. It was basically proved by P. H. Rabinowitz [7], [8]. However, in the case of relatively large external force, the numerical computations suggest not only the non-uniqueness of T-periodic solution, but also the existence of $2T$-periodic solution. In order to investigate these phenomena, we give one-parameter family of external force $\{f_\lambda\}_{\lambda \geq 0}$, where $f_0 = 0$, and consider the structure of periodic solution in the product space $\lambda \times u$. Here, λ is a positive parameter which somewhat represents the magnitude of external force. Then, as λ increase from 0, various bifurcation phenomena are observed by numerical computations. In particular, we can observe the period-doubling bifurcations which are known as very important phenomena along the route toward a so called "Chaos". However, for the nonlinear dissipative wave equation (1.1), there has been no rigorous proof on the existence of these bifurcation phenomena.

In order to attack this basic problem, we take the following strategy. At first, we construct a specific one parameter family of functions $\{u_\lambda\}_{\lambda>0}$. Next,

Communicated by Prof. T. Nishida, October 12, 1999
Revised August 11, 2000